the new science

Max Planck

the new science

Where Is Science Going?

The Universe in the Light of Modern Physics

The Philosophy of Physics

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these latter belongs our Planck. And that is why we love him.

I am quite aware that this clearance would mean the driving away of many worthy people who have built a great portion, and even perhaps the greatest portion, of the temple of science. But at the same time it is obvious that if the men who have devoted themselves to science consisted only of the two categories I have mentioned, the edifice could never have grown to its present proud dimensions, no more than a forest could grow if it consisted only of creepers.

But let us forget them. *Non ragionam di lor*. And let us fix our gaze on those who have found favor with the angel. For the most part they are strange, taciturn and lonely fellows. Yet, in spite of this mutual resemblance, they are far less like one another than those whom our hypothetical angel has expelled.

What has led them to devote their lives to the pursuit of science? That question is difficult to answer and could never be answered in a simple categorical way. Personally, I am inclined to agree with Schopenhauer in thinking that one of the strongest motives that lead people to give their lives to art and science is the urge to flee from everyday life, with its drab and deadly dullness, and thus to unshackle the chains of one's own transient desires, which supplant one another in an interminable succession so long as the mind is fixed on the horizon of daily environment.

But to this negative motive a positive one must be added. Human nature always has tried to form for itself a simple and synoptic image of the surrounding world. In doing this it tries to construct a picture which will give some sort of tangible expression to what the human mind sees in nature. That is what the poet does, and the painter, and the speculative philosopher and the natural philosopher, each in his own way. Within this picture he places the center of gravity of his own soul, so that he will find in it the rest and equilibrium which he cannot find within the narrow circle of his restless personal reactions to everyday life.

Among the various pictures of the world which are formed by the artist and the philosopher and the poet, what place does the world picture of the theoretical physicist occupy? Its chief quality must be a scrupulous correctness and internal logical coherence, which only the language of mathematics can express. On the other hand, the physicist has to be severe and self-denying in regard to the material he uses. He has to be content with reproducing the most simple processes that are open to our sensory experience, because the more complex processes cannot be represented by the human mind with the subtle exactness and logical sequence which are indispensable for the theoretical physicist.

Even at the expense of completeness, we have to secure purity, clarity, and accurate correspondence between the representation and the thing represented. When one realizes how small a part of nature can thus be comprehended and expressed in an exact formulation, while all that is subtle and complex has to be excluded, it is only natural to ask what sort of attraction this work can have? Does the result of such self-denying selection deserve the high-sounding name of world picture?

I think it does; because the most general laws on which the thought structure of theoretical physics is built have to be taken into consideration in studying even the simplest events in nature. If they were fully known, one ought to be able to deduce from them, by means of purely abstract reasoning, the theory of every process of nature, including that of life itself. I mean *theoretically*, because in practice such a process of deduction is entirely beyond the capacity of human reasoning. Therefore, the fact that in science we have to be content with an incomplete picture of the physical universe is not due to the nature of the universe itself but rather to us.

Thus the supreme task of the physicist is the discovery of the most general elementary laws from which the world picture can be deduced logically. But there is no logical way to the discovery of these elemental laws. There is only the way of intuition, which is helped by a feeling for the order lying behind the appearance, and this *Einfühlung* is developed by experience. Can one therefore say that any system of physics might be equally valid and possible? Theoretically there is nothing illogical in that idea. But the history of scientific development has shown that of all thinkable theoretical structures a single one has at each stage of advance proved superior to all the others.

It is obvious to every experienced researcher that the theoretical system of physics is dependent upon and controlled by the world of sense perception, though there is no logical way whereby we can proceed from sensory perception to the principles that underlie the theoretical structure. Moreover, the conceptual synthesis which is a transcript of the empirical world may be reduced to a few fundamental laws on which the whole synthesis is logically built. In every important advance the physicist

finds that the fundamental laws are simplified more and more as experimental research advances. He is astonished to notice how sublime order emerges from what appeared to be chaos. And this cannot be traced back to the workings of his own mind but is due to a quality that is inherent in the world of perception. Leibniz well expressed this quality by calling it a pre-established harmony.

Physicists sometimes reproach the philosophers who busy themselves with theories of knowledge, claiming that the latter do not appreciate this fact fully. And I think that this was at the basis of the controversy waged a few years ago between Ernst Mach and Max Planck. The latter probably felt that Mach did not fully appreciate the physicist's longing for perception of this pre-established harmony. This longing has been the inexhaustible source of that patience and persistence with which we have seen Planck devoting himself to the most ordinary questions arising in connection with physical science, when he might have been tempted into other ways which led to more attractive results.

I have often heard that his colleagues are in the habit of tracing this attitude to his extraordinary personal gifts of energy and discipline. I believe they are wrong. The state of mind which furnishes the driving power here resembles that of the devotee or the lover. The long-sustained effort is not inspired by any set plan or purpose. Its inspiration arises from a hunger of the soul.

I am sure Max Planck would laugh at my childish way of poking around with the lantern of Diogenes. Well, why should I tell of his greatness? It needs no paltry more right than he himself could have anticipated. He could not foresee that our whole concept of the structure of matter would be based on the quantum theory; neither could he have had any presentiment of the future development of quantum mechanics, quantum electrodynamics, etc., which may still have the greatest surprises in store for the physicists.

Planck, who revolutionized science because he had to do so, became a kind of godfather to the second great revolution which shook the fundament of our science during his lifetime—Einstein's theory of relativity. He was one of the very first advocates of Einstein's ideas, and it was he who was responsible for Einstein's coming to Berlin and accepting a position at the academy there and at the Kaiser Wilhelm Institute. Planck was fascinated by Einstein's theory in all its phases, and he understood its implications much earlier than most of his contemporaries.

According to his own statement, Planck was especially attracted to the theory of relativity because of its content of absolutivity, meaning the unique position ascribed to the velocity of light. But that remark of Planck's was certainly not meant to be a sufficient explanation of his special interest in the theory of relativity. Revolutionary as it is in its introduction of entirely new concepts of space and time and in the unrelenting strictness with which statements sanctified by tradition were recognized as meaningless and cast away, still the theory of relativity is the keystone, missing for centuries, which perfected the structure of classical physics. That, the writer believes, is the real reason for Planck's strong affection for Einstein's work.

forward in the name of science would be almost sure to find believers and disciples somewhere or other.

In the midst of this confusion it is natural to ask whether there is any rock of truth left on which we can take our stand and feel sure that it is unassailable and that it will hold firm against the storm of skepticism raging around it. Science, in general, presents us with the spectacle of a marvelous theoretical structure which is one of the proudest achievements of constructive reasoning. The logical coherence of the scientific structure was hitherto the object of unstinted admiration on the part of those who criticized the fundamentals of art and religion. But this logical quality will not avail us now against the skeptics' attack. Logic in its purest form, which is mathematics, only co-ordinates and articulates one truth with another. It gives harmony to the superstructure of science; but it cannot provide the foundation or the building stones.

Where shall we look for a firm foundation upon which our outlook on nature and the world in general can be scientifically based? The moment this question is asked the mind turns immediately to the most exact of our natural sciences, namely, physics. But even physical science has not escaped the contagion of this critical moment of history. It is not merely that the claim to reliability put forward by physical science is questioned from the outside; but even within the province of this science itself the spirit of confusion and contradiction has begun to be active. And this spirit is remarkably noticeable in regard to questions that affect the very fundamental problem of how far and in what way the human mind is capable of coming to a knowledge of external reality. To take one

And thus we strike out the positivist *als-ob* (As-If) and attribute a higher kind of reality than that of mere description of immediate sensory impressions to the practical discoveries that have been already mentioned—Faraday's, etc. Once we take this step we lift the goal of physical science to a higher level. It is not restricted to the mere description of bare facts of experimental discovery; but it aims at furnishing an ever increasing knowledge of the real outer world around us.

At this point a new epistemological difficulty enters. The basic principle of the positivist theory is that there is no other source of knowledge except within the restricted range of perception through the senses. Now there are two theorems that form together the cardinal hinge on which the whole structure of physical science turns. These theorems are: (1) *There is a real outer world which exists independently of our act of knowing*; and (2) *The real outer world is not directly knowable.* To a certain degree these two statements are mutually contradictory. And this fact discloses the presence of an irrational or mystic element which adheres to physical science as to every other branch of human knowledge. The knowable realities of nature cannot be exhaustively discovered by any branch of science. This means that science is never in a position completely and exhaustively to explain the problems it has to face. We see in all modern scientific advances that the solution of one problem only unveils the mystery of another. Each hilltop that we reach discloses to us another hilltop beyond. We must accept this as a hard-and-fast irrefutable fact. And we cannot remove this fact by trying to fall back upon a basis which would restrict the scope of science from the material that the physicist has to work upon. And the first process which this raw material must undergo is one of elimination and refinement. From the whole complex of sensory data everything must be cut away and discarded which may have arisen from the subjective constructive tendencies of the sensory organs themselves. And, furthermore, everything must be eliminated which can be attributed to the accident of special circumstances. In this latter connection attention must be paid to the fact that measuring instruments may affect the results that are being arrived at during the process of observation. That is all the more likely to be the case in the observation of minutiae.

Supposing all the above conditions to have been verified, then the physicist's picture of the external universe has only one further requirement to fulfill. Throughout its whole composition it must be free from everything in the nature of a logical incoherence. Otherwise the researcher has an entirely free hand. He may give rein to his own spirit of initiative and allow the constructive powers of the imagination to come into play without let or hindrance. This naturally means that he has a significant measure of freedom in making his mental constructions; but it must be remembered that this freedom is only for the sake of a specific purpose and is a constructive application of the imaginative powers. It is not a mere arbitrary flight into the realms of fancy.

The physicist is bound, by the very nature of the task in hand, to use his imaginative faculties at the very first step he takes. For the first stage of his work must be to take the results furnished by a series of experimental measurements and try to organize these under one law.

casioned by the alloy could be detected by putting the crown, and equal weights of gold and silver, separately in a vessel of water and measuring the difference of the overflow. Newton noticed the movement of an apple falling from a tree in his orchard and he connected that observation with the motion of the moon in relation to the earth. Einstein observed the state of a gravitating body in a fixed box, and considered this in juxtaposition with the state of a body free from gravitation in a box subjected to a process of upward acceleration. Niels Bohr associated the orbital rotation of the electrons around the nucleus of an atom with the movement of planets around the sun. All these combinations were productive of famous results. Indeed it would be an interesting mental exercise if one were to take as many as possible of the hypotheses which have proved significant of results in the pursuit of physical science and then try to discover the respective combinations of ideas to which the hypotheses owed their origin. But the task would be a difficult one because, generally speaking, creative master minds have felt a personal aversion from the idea of unfolding before the public gaze those delicate threads of thought out of which their productive hypotheses were woven, and the myriad other threads which failed to be interwoven into any final pattern.

The utility of an hypothesis, once it has been put forward, can be tested only by following out the logical results that flow from its application. This has to be done in a purely logical—and primarily mathematical—way, whereby the hypothesis is used as a starting point and as complete a theoretical system as possible is developed from it. Once the theoretical system has thus been fully every event in every instance there must be a corresponding cause? Would the thought involve a logical contradiction that in this or that case the event has absolutely happened of itself and has no causal relation whatsoever to any other event? Of course the answer is in the negative; for it is very easy to *think* of an event as having no explanatory cause whatsoever. In such cases we speak of miracles and wonders and magic. And the simple fact that there exists a whole range of literature whose scenes are laid in wonderland is proof in itself that the concept of strict causality is not an inherent necessity of human thought. Indeed the human mind finds little difficulty in thinking of everything in the world as turning topsy-turvy. We can say to ourselves that tomorrow the sun may rise in the west, for a change. We can say to ourselves that a miracle of nature may occur, contrary to all the known laws of nature. We can think of the Niagara Falls for instance as shooting upwards, though this would be impossible in the world of reality. I can think of the door of my room in which I am now writing as opening of its own accord. And I can think of historical personages as entering the room and standing beside my table. In the world of reality to talk of such events may be meaningless and we may call them impossible, at least in our everyday way of reasoning. But we must distinguish this kind of impossibility from a logical impossibility, such as the idea of a square circle or that the part of something is greater than the whole, for no matter what efforts we make to think such things we cannot think them, inasmuch as they entail an inner contradiction. We can think of a part and we can think of the whole to which it belongs but we cannot think of the part as greater than

the whole. This kind of impossibility is inherent in the nature of human thought itself, whereas the idea of something happening outside the range of causation is quite logically coherent.

Thus from the outset we can be quite clear about one very important fact, namely, that the validity of the law of causation for the world of reality is a question that cannot be decided on grounds of abstract reasoning. But reality, no matter what may be said to the contrary, is only a particular and small section of that immense sphere over which human thought can range. This is true even though our powers of imagination have always to take their cue from some real experience. Indeed experience is for us the starting point of all thought; but we possess the gift of going beyond reality in thought. And were it not for this faculty of the imaginative intellect we should have no poetry and no music and no art. Indeed it is one of the highest and most precious gifts that man possesses, this power of lifting himself in thought into the realms of light whenever the weight of everyday life presses upon him and makes itself intolerable.

The creations of art are similar to those of science at least to the extent that scientific research, in the strictest sense of the term, could never advance without the creative force of the imaginative intellect. The man who cannot occasionally imagine events and conditions of existence that are contrary to the causal principle as he knows it will never enrich his science by the addition of a new idea. And this power of thinking beyond the range of causation is a prerequisite not only for the construction of hypotheses but also for the satisfactory co-ordination of results that have been arrived at through scientific important role in theoretical science. They are employed as very useful instruments of thought in the carrying out of researches and the construction of theories. Therefore they certainly do not involve any contradiction of the laws of thought itself.

Once we have decided that the law of causality is by no means a necessary element in the process of human thought, we have made a mental clearance for the approach to the question of its validity in the world of reality. Now in the first place let us ask what is meant by the term "causation"? We might mean by it a regular interrelation between effects that follow one another in time. But we can at once ask whether this relation be founded in the nature of things themselves, or is it totally, or partly, a product of the imaginative faculty? Might it not be that mankind originally developed this concept of causation to meet the necessities of a practical life, but afterward found that if men were to confine themselves to an outlook exclusively based on this principle life would then turn out to be unbearable? We need not delay here to discuss the various philosophical aspects of these questions. For our present purpose it is much more important to ask whether the causal connection between events must be considered as absolutely complete and always unbroken or are there events in the world which do not enter the chain as connecting links?

Let us first see whether this question can be settled by a systematic application of deductive reasoning. As a matter of fact, some of the most famous philosophers in the history of human thought have produced solutions of the causal problem which were based on purely abstract grounds. They took their first stand on the axiom *ex nihilo*

nihil fit, that nothing comes from nothing, in other words that no event in the world holds in itself an adequate explanation of its own existence. Reasoning back from this standpoint the philosophers of what is generally called the rationalist school established as a logical necessity the existence of a Supreme Cause. This Supreme Cause is the God of Aristotle and the Scholastic philosophers. As a logical consequence of the line of reasoning thus adopted it was necessary to attribute to this Godhead the possession in their plenitude of all the perfections that are present in the world. If there be an actually existent Supreme Cause outside of the world, who is the Creator of the world and the Creator of all things in the world, then man can deduce the nature of this Supreme Cause only through a study of His handiwork. From this one can easily see that the nature to be attributed to that Supreme Cause must necessarily depend upon man's outlook on created things. In other words the concept of the Divinity in this case must take its color from the world outlook either of the individual philosopher in question or of the particular cultural background to which he belongs. In the attempt which the Scholastics made to harmonize the Jehovah of the Jewish culture with the rational God of Aristotle, emphasis was laid on the fact that there is no logical contradiction whatsoever in the idea of the Creator interposing his hand suddenly within the order of His own creation, and thus we have belief in miracles and wonders established on a philosophical basis. Therefore in the philosophy of the historic rationalist school, though the order of nature is admitted as inevitably predetermined by the Supreme Cause, yet the causal chain in the world itself may at any time be inpenings to certain causes. But at the same time this force of habit cannot explain why we should make the attribution at all. In Fritz Reuter's story *Rei's Nah Belligen,* the peasants undoubtedly made a ludicrous mistake in supposing that there were horses concealed in the steam engine, just as the ancient Greek peasant made a mistake in attributing the thunder to the personal anger of Zeus. But this is not the point here. The point rather is to answer the question why these events should be attributed to a cause at all and how it is that the concept of causation itself arises when we see one event following another. The mere regular succession of impressions does not explain this.

If we go a little deeper into the consideration of the empiricist theory and ask where it would finally lead us were we to pursue it to its logical consequences we shall thus be putting it to a practical test. In the first place we must bear in mind the fact that when there is question of sensory perception as the sole and exclusive source of knowledge, then there can be question only of each one's personal sensory perception in each one's own consciousness. That other men have similar perceptions we can assume only by analogy; but, on the empiricist theory, we cannot know this nor can we logically prove it. Therefore if we are to abide by the logical consequences of the empirical doctrine and exclude all arbitrary assumption, we must confine ourselves, each one of us, to the grounds of his or her own personal sense perceptions. Then the principle of causation is only a framework for our experiences, connecting them with one another as they enter through the senses, and being entirely unable to tell us anything of what is to come next, it cannot tell us

whether the sequence of our experiences may not be broken in a moment. This condition of affairs would seem to obliterate every line of distinction between the sensory perceptions arising from the world of ordinary happenings and those that have no foundation whatsoever in that world. Take the case of sleep, for instance. I may dream all sorts of things during the night; but the moment I wake up the reality of my surroundings gives the lie to the dream. The empiricist, however, cannot logically admit that. For him there is no waking reality; because the subjective sensation is the sole source of awareness in consciousness and is the sole basis and criterion of knowledge. Now the dreamer during the dream believes automatically in its reality and, according to the empiricists, the wide-awake person believes automatically in the reality of his sense perceptions; but has no more reason than the dreamer has for saying that one set of perceptions is false and the other true.

On the grounds of pure logic, of course, this system of thought, which is commonly called solipsism, is impregnable. The solipsist establishes his ego at the center of creation, and he does not consider any knowledge as real or sound except that which he for the moment is receiving through his sensory perception. Everything else is derivative and secondary. When the solipsist goes to sleep at night the world ceases to exist for him the moment his eyes and ears and sense of smell and touch become inactive. On rising in the morning everything is new to him again. Here of course I am only imagining what a human being would be if he were a logical consequence of the empirical teaching.

All this of course amounts to a repudiation of common sense; so much so that even the most advanced skeptics of this school find themselves constantly compromising between the claims of common sense and the purely logical conclusions of their own philosophic system. In this connection it is interesting to call attention for a moment to the figure of one of the most outstanding personalities in the subjectivist school, namely Bishop Berkeley. As a student Berkeley studied Locke. But he was of a very deep religious nature and launched a strong criticism against Locke's philosophy because of its skepticism. For Berkeley all things exist only in the mind and the external world can be accounted for only by saying that it exists in the mind of God. He arrives at the existence of God in this way: There are in our own consciousness impressions which are independent of our own wills and sometimes exist even contrary to our wishes. For these impressions we must seek a cause elsewhere than in ourselves and so Berkeley is led to establish the existence of God by practically the same line of reasoning as the rationalist school. For him, however, mind and mind alone exists—the Divine Mind and the human mind. The world of reality as we perceive it exists only in our own mind. Therefore with Berkeley we have no right to talk about a causal interrelation between things in the outer world of reality.

To sum up, empiricism is unassailable on the fundamental ground of a pure logic; and its conclusions are equally impregnable. But if we look at it purely from the viewpoint of knowledge it leads into a blind alley, which is called solipsism. In order to escape from this *impasse* there is no other way open but to jump the wall at some part of it, and preferably at the beginning. This mation of our previous conviction, namely, that the nature and universal validity of the law of causation cannot be definitely decided upon any grounds of purely abstract reasoning. The transcendental and positivist viewpoints are irreconcilable and they will remain so as long as the race of philosophers lasts.

If pure reasoning had the last word in dealing with such cases then the outlook would be hopeless for any satisfactory settlement of the causative problem. But philosophy, after all, is only one branch of human activity in the study of problems effecting nature and mankind. Science is another branch. And where philosophy has failed in a given instance we are perfectly justified in turning to science and asking whether it may not have a satisfactory answer to suggest.

Now, let us first ask whether the various branches of science are divided against one another on this question of causation, just as philosophy is divided? At the very threshold of this inquiry it may be objected that a problem which falls within the scope of philosophy and which philosophy fails to solve cannot possibly be solved within the limits of a single science. This objection is urged on the grounds that philosophy furnishes the mental foundations on which scientific investigation rests. Philosophy must precede every special science and we should be going against the grain of our whole mental discipline if one of the special sciences were to take up the treatment of general philosophic questions.

That argument is very often urged. But in my opinion the weakness of it is that it leaves out of consideration the collaboration which actually exists between philosophy and the various special sciences. We must rememical or epistemological foundations on which it works. Does science as a matter of fact occupy itself exclusively with data immediately given by sensory impressions and their systematic organization according to laws of reason? Or does it at the very outset of its activities reach out beyond the knowledge given us by this immediate source and make, as it were, a jump into the metaphysical sphere?

I do not think that there can be any doubt whatsoever as to the answer. The first alternative is ruled out and the second affirmed in the case of each special science. Indeed it may be said that every individual science sets about its task by the explicit renunciation of the egocentric and anthropocentric standpoint. In the earlier stages of human thought mankind turned its attention exclusively to the impressions received through the senses, and primitive man made himself and his own interests the center of his system of reasoning. Confronted with the powers of nature around him, he thought that they were animated beings like himself and he divided them into two classes, the one friendly and the other inimical. He divided the plant world into the categories of poisonous and nonpoisonous. He divided the animal world into the categories of dangerous and harmless. As long as he remained bound within the limits of this method of treating his environment it was impossible for him to make any approach towards real scientific knowledge. His first advance in this knowledge was accomplished only after he had taken leave of his own immediate interests and banished them from his thought. At a later stage he succeeded in abandoning the idea that the planet whereon he lives is the central point of the universe. Then he took

up the more modest position of keeping as far as possible in the background, so as not to intrude his own idiosyncrasies and personal ideas between himself and his observations of natural phenomena. It was only at this stage that the outer world of nature began to unveil its mystery to him, and at the same time to furnish him with means which he was able to press into his own service and which he could never have discovered if he had continued looking for them with the candlelight of his own egocentric interests. The progress of science is an excellent illustration of the truth of the paradox that man must lose his soul before he can find it. The forces of nature, such as electricity, for instance, were not discovered by men who started out with the set purpose of adapting them for utilitarian purposes. Scientific discovery and scientific knowledge have been achieved only by those who have gone in pursuit of it without any practical purpose whatsoever in view. The few examples that I have mentioned make this abundantly clear. Heinrich Hertz, for instance, never dreamt that his discoveries would have been developed by Marconi and finally evolved into a system of wireless telegraphy. And Roentgen could never have called up a vision of the immense range of beneficial purposes to which the X-rays are applied today.

I have said that the first step which every specialized branch of science takes consists of a jump into the region of metaphysics. In taking this jump the scientist has confidence in the supporting quality of the ground whereon he lands, though no system of abstract reasoning could have previously assured him of that. In other words, the fundamental principles and indispensable postulates of every genuinely productive science are not based on pure logic but rather on the metaphysical hypothesis—which no rules of logic can refute—that there exists an outer world which is entirely independent of ourselves. It is only through the immediate dictate of our consciousness that we know that this world exists. And that consciousness may to a certain degree be called a special sense. And one may go even so far as to say that the existence of the exterior world strikes the consciousness of each individual in some particular way. It is as if we looked at some distant object through a pair of glasses and as if each one were wearing glasses of a slightly different shade of color. And we must take this into account when we deal scientifically with natural phenomena. The first and most important quality of all scientific ways of thinking must be the clear distinction between the outer object of observation and the subjective nature of the observer.

Once the scientist has begun by taking his leap into the transcendental he never discusses the leap itself nor worries about it. If he did science could not advance so rapidly. And anyhow—which is fundamentally a consideration of no less importance—this line of conduct cannot be refuted as inconsistent on any logical grounds.

Of course there is the positivist theory that man is the measure of all things. And that theory is irrefutable in so far as nobody can object on logical grounds to the action of a person who measures all things with a human rule, and resolves the whole of creation ultimately into a complex of sensory perceptions. But there is another measure also, which is more important for certain problems and which is independent of the particular method

and nature of the measuring intellect. This measure is identical with the *thing* itself. Of course it is not an immediate datum of perception. But science sets out confidently on the endeavor finally to know the *thing* in itself, and even though we realize that this ideal goal can never be completely reached, still we struggle on toward it untiringly. And we know that at every step of the way each effort will be richly rewarded. The history of science is at hand to confirm our faith in this truth.

Having once assumed the existence of an independent external world, science concomitantly assumes the principle of causality as a concept entirely independent of sense perception. In applying this principle to the study of natural phenomena science first investigates if and how far the law of causal relation is applicable to the various happenings in the world of nature and in the realm of the human spirit. Science finds itself here exactly on the same footing which Kant took as the starting point of his theory of knowledge. As in the case of Kantian philosophy, so also in the case of each special branch of science the causal concept is accepted at the outset as belonging to those categories without which no progress in knowledge can be made. But we must make a certain differentiation here. Kant took not merely the concept of causality but also to a certain degree the meaning of the causal law itself as an immediate datum of knowledge and therefore universally valid. Specialized science cannot go thus far. It must rather confine itself to the question as to what significance the law of causality can be proved to have in each individual case, and thus through research give practical meaning and value to the empty framework of the causal concept.

5 / Causation and Free Will

The Answer of Science

We now come to ask whether and how far science can help us out of the obscure wood wherein philosophy has lost its way. What is the practical attitude adopted by the special sciences in regard to the universal and invariable validity of the law of causation? Does science in its everyday investigations accept the principle of causation as an indispensable postulate? Does it act upon the assumption that there are no loopholes in the causally governed order of nature? Or, while using the principle as a working hypothesis, does scientific practice intimate that there are certain happenings in nature where the law of causation does not function, and that there are regions in the mental sphere where the causal writ does not run? In our endeavor to find a definite answer to those questions we shall have to put them singly to each of the several branches of specialized science. In doing this, of course, we shall have to be content with quite a summary cross-examination. What has physical science to say to

our problem? What has the science of biology to answer? And what have the humanist sciences, such as psychology and history, to say?

Let us begin with the most exact of the natural sciences, namely, physics. In classical dynamics, among which we must include not only mechanics and the theory of gravitation but also the Maxwell-Lorentz view of electrodynamics, the law of causality has been given a formulation which for exactitude and strictness may be considered almost as ideal, even though it may be somewhat one-sided. It is expressed in a system of mathematical equations through which all happenings in any given physical picture can be absolutely predicted if the time and space conditions are known—that is to say, if the initial state be known and the influences which are brought to bear upon the picture from outside. To put the matter in a more concrete way: according to the law of causation as expressed in the equations of classical dynamics, we can tell where a moving particle or system of particles may be located at any given future moment if we know their location and velocity now and the conditions under which the motion takes place. In this way it was made possible for classical dynamics to reckon beforehand all natural processes in their individual behavior and thus to predict the effect from the cause. The last significant advance which classical dynamics achieved in our day came about through the general relativity theory of Einstein. This theory welded together Newtonian gravitation and Galileo's law of inertia. Several attempts have been made recently to show that the relativity theory corroborates the positivist attitude and in a certain sense is incompatible with transcendental philosophy. These attempts are entirely mistaken. For the foundation of the relativity theory is not based on the rule that all time and space dimensions have only a relative meaning, which is determined by the reference system of the observer. The foundation of the relativity theory lies in the fact that in the four-dimensional space-time manifold there is a measure, namely the distance between two points approximating with infinite closeness. This is the so-called tensor or *Massbestimmung*, which for all measuring observers and for all reference systems has the self-same value, and it therefore is of a transcendental character entirely independent of any arbitrary action of the human will.

Into this harmonized system of classical-relativist physics, however, the quantum hypothesis has recently introduced a certain disturbance, and one cannot yet definitely say what influence the subsequent development of the hypothesis may have on the formulation of fundamental physical laws. Some essential modification seems to be inevitable; but I firmly believe, in company with most physicists, that the quantum hypothesis will eventually find its exact expression in certain equations which will be a more exact formulation of the law of causality.

Besides dynamical laws applied to individual cases, physical science recognizes other laws also, which are called statistical. These latter express to a fairly accurate degree the probability of certain happenings occurring and therefore they allow for exceptions in particular cases. A classical example of this is the conduction of heat. If two bodies of different temperatures be brought into contact with one another then, according to the two laws of thermodynamics, the heat energy will always through innumerable series of particular processes which are independent of one another and which we call molecular movements. And investigation has further shown that if we presuppose the validity of dynamical laws for each of these particular happenings—that is to say, the law of strict causality—then we can arrive at the causal results through this type of observation. In point of fact, statistical laws are dependent upon the assumption of the strict law of causality functioning in each particular case. And the nonfulfillment of the statistical rule in particular cases is not therefore due to the fact that the law of causality is not fulfilled, but rather to the fact that our observations are not sufficiently delicate and accurate to put the law of causality to a direct test in each case. If it were possible for us to follow the movement of each individual molecule in this very intricate labyrinth of processes, then we should find in each case an exact fulfillment of the dynamical laws.

In speaking of physical science under this aspect we must always distinguish between two different methods of research. One is the macroscopic method, which deals with the object of research in a general and summary manner. The other is the microscopic method, which is more delicate and detailed in its procedure. It is only for the macroscopic observer—that is to say, the man who deals with big quantities in a wholesale way—that chance and probability exist in regard to single elements in the object that he handles. The extent and importance of the chance elements are of course dependent on the measure of knowledge and skill which is brought to bear on the object. On the other hand, for the microscopic investigator only accuracy and strict causality exist. His to nature's hand. As the crop grew, the letters that corresponded to the manured furrows showed rows of clover much taller and more luxuriant than that in the other parts of the field; so that the passers-by were able to read the sentence: "This part has been manured with gypsum." History does not relate whether the obstinate peasants were or were not convinced by the proof. But that is neither here nor there; for nobody can be forced on purely logical grounds to acknowledge the causal connection, because the causal connection is not logically demonstrable. The point of the illustration here is that, if in a particular case we introduce a cause which of its very nature "flows into" the result, as the Scholastics used to say, and if the result is in full accord with what was predicted, then we can be certain of the causal relation. In the instance of Franklin's clover there could possibly be no other explanation except that of the manuring, and this explanation, as a cause, has a natural and exclusive connection with the result.

Of course it may be said that the law of causality is only after all an hypothesis. If it be an hypothesis, it is not an hypothesis like most of the others, but it is a fundamental hypothesis because it is the postulate which is necessary to give sense and meaning to the application of all hypotheses in scientific research. This is because any hypothesis which indicates a definite rule presupposes the validity of the principle of causation.

We now come to those sciences which deal with human events. Here the method which the scientist follows can have nothing like the same exactitude as that which he follows in physics. The object of his study is the human mind and its influence on the course of events. The great asked above. One may have opinions and make suppositions and assumptions; but these do not furnish logical grounds for an answer. Still I think that it may be said definitely that the direction in which the humanist sciences, such as psychology and history, are developing nowadays furnishes certain grounds for presuming that the question should be answered in the affirmative. The part which force plays in nature, as the cause of motion, has its counterpart in the mental sphere in motive as the cause of conduct. Just as at each and every moment the motion of a material body results necessarily from the combined action of many forces, so human conduct results with the same necessity from the interplay of mutually reinforced or contradicting motives, which partly in the conscious and partially also in the unconscious sphere work their way forward toward the result.

Of course it is perfectly true that many acts which are done by human beings appear to be inexplicable. At times it is an extraordinarily difficult riddle to find anything like reasonable grounds for certain acts, and other acts seem so utterly foolish as to suggest no grounds at all. But consider for a moment the way these acts appear to a trained psychologist and the way they appear to the ordinary man in the street. What is entirely puzzling to the latter is often quite clear to the former. Therefore if we could study the acts of the human being at very close and intimate quarters, we should find that they can be accounted for through causes which lie in the character or in the momentary emotional tension or in the specific external environment. And in those cases where it is extremely difficult and well-nigh impossible to discover these explanatory causes, then we have at least grounds for assuming that if we cannot find any motive as an explanation, we must attribute this not actually to the absence of motive but rather to the unsatisfactory nature of our knowledge of the peculiarities of the situation. Here we have the same case as in the throwing of the unsymmetrical dice. We know that the way in which the dice finally comes to rest is the net result of all the factors active in the throwing of the dice, but in the case of a single throw we cannot detect the function of strict causality. And so, even though the motive of a certain line of human conduct may often lie utterly hidden, conduct entirely without motive is scientifically just as incompatible with the principles on which mental science is carried on as the assumption of absolute chance in inorganic nature is incompatible with the working principle of physical science.

It is not merely, however, that conduct is conditioned by the motives which lead to it. Each act has also a causal influence on subsequent behavior. And so in the interchange of motive and conduct we have an endless chain of events following one another in the spiritual life, in which every link is bound by a strict causal relation not only with the preceding link but also with the following one.

Attempts have been made to find a way to free these links from the causal chain. Hermann Lotze, in open contradiction to Kant, put forward the suggestion that such a causal chain can have no end, although it has a beginning. In other words, that circumstances occur in which motives appear entirely independently, not caused by any preceding influence, so that the conduct to which these motives lead will be the first link in a new chain.

Such an interpretation, Lotze held, must be given especially to the acts of those choice spirits that are called creative geniuses.

Even though we may not question the possibility of such cases happening in the world of reality, yet we may reasonably answer that the thoroughgoing scientific research which has been carried on in the region of psychology would have pointed to such a possibility. But as far as psychological research has gone, there are no indications which might furnish a starting ground for this theory of the so-called free beginning. On the contrary, the deeper scientific research goes into the peculiarities that have characterized even the great spiritual movements of world history, more and more the causal relation emerges into the open. The dependence of each event upon preceding fact and preparatory factors gradually begins to appear under the strong light of scientific investigation, so much so as to warrant the statement that present-day scientific procedure in psychology is founded practically exclusively on the principle of causal interrelations and the assumption of an active law of causality which permits no exceptions. This means that the postulate of complete determinism is accepted as a necessary condition for the progress of psychological research.

Under these circumstances it is obvious that we cannot erect a definite boundary and say: Thus far but no farther. The principle of causality must be held to extend even to the highest achievements of the human soul. We must admit that the mind of each one of our greatest geniuses—Aristotle, Kant or Leonardo, Goethe or Beethoven, Dante or Shakespeare—even at the moment of its highest flights of thought or in the most profound inner workings of the soul, was subject to the causal fiat and was an instrument in the hands of an almighty law which governs the world.

The average reader may be easily taken aback by such a statement. It may sound derogatory to speak thus of the creative achievements of the highest and noblest of the human race. But on the other hand it must be remembered that we ourselves are only common mortals, and that we could never hope to be in a position to follow out the delicate play of cause and circumstance in the soul of the genius. There is nothing derogatory in saying that they are subject to the law of cause and effect, though it would be derogatory of course if this were interpreted in the sense that the ordinary mortal is capable of following the workings of that law in the case of supremely gifted souls. Nobody would feel it disrespectful if one were to say that some superhuman intelligence could understand a Goethe or a Shakespeare. The whole point lies in the inadequacy of the observer. Just so the macroscopic physicist is entirely unable to pursue microscopic workings in natural phenomena, yet, as we have seen, this does not mean that the law of causality is not valid for these microscopic happenings.

Where is the sense then, it may here be asked, in talking of definite causal relations in regard to cases wherein nobody in the world is capable of tracing their function?

The answer to that question is simple enough. As has been said again and again, the concept of causality is something transcendental, which is quite independent of the nature of the researcher, and it would be valid

even if there were no perceiving subject at all. We shall see more clearly the inner meaning of the causal concept if we consider the following:

At this present moment of time and space the human intellect as we know it may possibly not be the highest type of intellect in existence. Higher intelligences may exist in other places or may appear in other epochs. And the intellectual level of these beings may be as much above ours as ours is above the protozoa. Then it may well happen that before the penetrating eye of such intelligences even the most fleeting moment of mortal thought, as well as the most delicate vibration in the ganglia of the human brain, could be followed in each case, and that the creative work of our mortal geniuses could be proved by such an intelligence to be subject to unalterable laws, just as the telescope of the astronomer traces the links of the manifold movement of the spheres.

Here, as everywhere else, we must differentiate between the validity of the causal principle and the practicability of its application. Under all circumstances the law of causation is valid, because of its transcendental character. But as its application can be carried out in full detail only by the microscopic observer in natural science, so in the region of the human mind the law can be applied only by an intelligence that is far superior to the object of research. The smaller the distance between the investigator and the object in this case, the more uncertain and fallible will be the causal and scientific treatment. The whole problem lies in the difficulty, indeed the impossibility, with which we are faced in trying to understand the behavior of a genius from the unobservant man in the street, yet educated as well as uneducated people often turn to the dim region of mystery for light on the ordinary problems of life. One would imagine that they would turn to science, and it is probably true that those who do so are more intensely interested in science and are perhaps greater in number than any corresponding group of people in former times; but still the fact remains that the drawing power of systems which are based on the irrational is at least as strong and as widespread as ever before, if not more so. The Monist League which was formed some years ago with so much *éclat* and promise, for the purpose of establishing a world outlook based on purely scientific grounds, has certainly not achieved any success corresponding to the rival systems.

How is this peculiar fact to be explained? Is there, in the last analysis, some basically sound foothold for this belief in miracle, no matter how bizarre and illogical may be the outer forms it takes? Is there something in the nature of man, some inner realm, that science cannot touch? Is it so that when we approach the inner springs of human action science cannot have the last word? Or, to speak more concretely, is there a point at which the causal line of thought ceases and beyond which science cannot go?

This brings us to the kernel of the problem in regard to free will. And I think that the answer will be found automatically suggested by the questions which I have just asked.

The fact is that there is a point, one single point in the immeasurable world of mind and matter, where science and therefore every causal method of research is

inapplicable, not only on practical grounds but also on logical grounds, and will always remain inapplicable. This point is the individual ego. It is a small point in the universal realm of being; but in itself it is a whole world, embracing our emotional life, our will and our thought. This realm of the ego is at once the source of our deepest suffering and at the same time of our highest happiness. Over this realm no outer power of fate can ever have sway, and we lay aside our own control and responsibility over ourselves only with the laying aside of life itself.

And yet there is a way in which the causal method can be applied within the limits of this inner realm. In principle there is no reason whatsoever why the individual should not make himself the observer of what has happened within himself. In other words, he can look back over the experiences through which he has passed and endeavor to link them up in their causal relations. There is no reason indeed, at least in principle, why he should not scrutinize each experience—by which I mean each decision and line of conduct which he has taken—and study it from the viewpoint of finding out the cause from which it resulted. Of course that is an extremely difficult task; but it is the only soundly scientific way of dealing with our own lives. In order to carry out this plan of action the facts of our own lives which we now place under observation would have to be distanced in the past, so that our present complex of living emotions and inclinations would not enter as factors into the observation. If we could possibly carry out the plan in this detached way, then each experience through which we have passed would make us immeasurably more intelligent

Freedom = inability of the eye to see itself!!

is a relative degree to which he might subject his own experiences to causal scrutiny; and I have mentioned this as illustrative of the general principle.

It will occur to many readers to ask if thus in relation to the chain of causality the freedom of the individual will, here and now, is only apparent and results solely from the defects of our own understanding. That way of putting the case is, I am convinced, entirely mistaken. We might illustrate the mistake by saying that it is like the mistake of suggesting that the inability of a runner to outrun his own shadow is due to his lack of speed. The fact that the individual here and now, in regard to his own living present act, cannot be subject to the law of causation is a truth that is based on a perfectly sound logical foundation of an a priori kind, such as the axiom that the part is never greater than the whole. The impossibility of the individual contemplating his own activity here and now under the light of the causal principle would hold good even in the case of the superintelligence postulated by Laplace. For, even though this superintelligence might be able to trace the causal structure in the achievements of the most gifted geniuses of the human race, yet that same superintelligence would have to renounce the idea of studying the activities of its own ego at the moment it contemplated the activities of our mortal ego. If there be a Supreme Wisdom whose celestial nature is infinitely elevated above ours, and who can see every convolution in our brains and hear every pulse beat of each human heart, as a matter of course such a Supreme Wisdom sees the succession of cause and effect in everything we do. But this does not in the least invalidate our own sense of responsibility for our own actions. From this standpoint we are on an equal footing with the saints and confessors of the most sublime religions. We cannot possibly study ourselves at the moment or within the environment of any given activity. Here is the place where the freedom of the will comes in and establishes itself, without usurping the right of any rival. Being emancipated thus, we are at liberty to construct any miraculous background that we like in the mysterious realm of our own inner being, even though we may be at the same time the strictest scientists in the world, and the strictest upholders of the principle of causal determinism. It is from this autarchy of the ego that the belief in miracles arises, and it is to this source that we are to attribute the widespread belief in irrational explanations of life. The existence of that belief in the face of scientific advance is a proof of the inviolability of the ego by the law of causation in the sense which I have mentioned. I might put the matter in another way and say that the freedom of the ego here and now, and its independence of the causal chain, is a truth that comes from the immediate dictate of the human consciousness.

And what holds good for the present moment of our being, holds good also for our own future conduct in which the influences of our present ego play a part. The road to the future always starts in the present. It is, here and now, part and parcel of the ego. And for that reason the individual can never consider his own future purely and exclusively from the causal standpoint. That is the reason why fancy plays such a part in the construction of the future. It is in actual recognition of this profound fact that people have recourse to the palmist and the

clairvoyant to satisfy their individual curiosity about their own future. It is also on this fact that dreams and ideals are based, and here the human being finds one of the richest sources of inspiration.

I might mention here in passing that this practical inapplicability of the law of causation extends beyond the individual. It extends to our relations with our fellow men. We are too much a part of the life of our fellow beings to be in a position to study them from the viewpoint of motives, which means the causal viewpoint. No ordinary human being can put himself in the position of the superintelligence imagined by Laplace and consider himself capable of tracing all the inner springs of action from which the conduct of his fellow men originates. On the other hand, however, I would mention here again a phase of the causal application corresponding to that which I have already spoken of in relation to the individual's capacity for scientifically observing his own past experience. To a relative degree it is possible to study the motives on which other people act, just as they are studied by the psychologist or the alienist. In all such cases there is to a certain degree the requisite distance between the researcher and the object of his research. And, therefore, to this extent there is no logical incoherence in the idea of a person studying the activities of his fellow beings. Indeed all who wish to influence others do so in everyday life, which is largely the secret of political success. It is the secret of all the power for good which so many people exercise in relation to their fellow beings. Most of us remember from childhood personalities whom we shirked because of some sort of innate feeling of insecurity in their presence, and on the other hand most of us, I imagine, have memories of acquaintances to whose influence we were willingly amenable because we felt a certain reverence toward them. And everybody is more or less familiar with the feeling of withdrawal which comes over one in the presence of a person who is suspected of seeing too clearly into the inner lives of others. All these immediate reactions bear witness to a sort of instinctive recognition that our own lives are, in the last analysis, subject to causation, though the ego as regards its immediate destiny cannot be subject to that law.

Science thus brings us to the threshold of the ego and there leaves us to ourselves. Here it resigns us to the care of other hands. In the conduct of our own lives the causal principle is of little help; for by the iron law of logical consistency we are excluded from laying the causal foundations of our own future or foreseeing that future as definitely resulting from the present.

But mankind has need of fundamental postulates for the conduct of everyday existence, and this need is far more pressing than the hunger for scientific knowledge. A single deed often has far more significance for a human being than all the wisdom of the world put together. And therefore there must be another source of guidance than mere intellectual equipment. The law of causation is the guiding rule of science; but the categorical imperative—that is to say, the dictate of duty—is the guiding rule of life. Here intelligence has to give place to character, and scientific knowledge to religious belief. And when I say religious belief here I mean the word in its fundamental sense. And the mention of it brings us to that much discussed question of the relation

between science and religion. It is not my place here nor within my competency to deal with that question. Religion belongs to that realm that is inviolable before the law of causation and therefore closed to science. The scientist as such must recognize the value of religion as such, no matter what may be its forms, so long as it does not make the mistake of opposing its own dogmas to the fundamental law upon which scientific research is based, namely the sequence of cause and effect in all external phenomena. In conjunction with the question of the relations between religion and science, I might also say that those forms of religion which have a nihilist attitude to life are out of harmony with the scientific outlook and contradictory to its principles. All denial of life's value for itself and for its own sake is a denial of the world of human thought, and therefore, in the last analysis, a denial of the true foundation not only of science but also of religion. I think that most scientists would agree to this, and would raise their hands against religious nihilism as destructive of science itself.

There can never be any real opposition between religion and science; for the one is the complement of the other. Every serious and reflective person realizes, I think, that the religious element in his nature must be recognized and cultivated if all the powers of the human soul are to act together in perfect balance and harmony. And indeed it was not by any accident that the greatest thinkers of all ages were also deeply religious souls, even though they made no public show of their religious feeling. It is from the co-operation of the understanding with the will that the finest fruit of philosophy has arisen, namely the ethical fruit. Science enhances the molecule of oxygen, whereas the molecule of hydrochloric acid is made up of half a molecule of chlorine and half a molecule of hydrogen. Therefore from the molecular weight we come to the atomic weight of an element as the smallest fraction which is found in a combination of elements. This atomic weight expresses the relative weights of each species of matter.

Although in Avogadro's law the concept of atomic weight has a certain absolute significance, at the same time it has quite a relative connotation. The Avogadrian atomic weight is only a relative number. Therefore it cannot be determined except by an arbitrary reference to the atomic weight of some special element or other, such as Hydrogen = 1 or Oxygen = 16. Without reference to some such given term, the number describing the atomic weight would have no meaning. Therefore it has for a long time been the aim of chemical researchers to free the concept of atomic weight from this restriction and try to give it a wider and more absolute meaning. This problem, however, is not very important for the practical chemist; because in the chemical analysis of substances there is always the question of relative proportions among the combining elements.

In every science it occasionally happens that there arises a conflict between two classes of people whom I may designate respectively as purists and pragmatists. The former strive always after a perfect co-ordination of the accepted axioms of their science, submitting them to an ever more and more rigid analysis, for the purpose of eliminating every contingent and foreign element. On the other hand, the pragmatists try to amplify the accepted first principles by the introduction of new ideas radiation of the zero point. It cannot be detected in the observation of ordinary processes because its streams through all bodies equally, just as the pressure of the atmosphere represents a very important force which plays no part in most of the movements that we observe, because the pressure is equal in all directions. Such a radiation hypothesis is perfectly reasonable, and its validity can be decided upon only by the question of what results follow from its application. For this application, however, it is absolutely necessary to furnish a special reference system that is immobile, namely that in which the zero radiation is equal in all directions. Through the absolute energy of the neutral field the absolute energy of every other electromagnetic field is thereby established.

Coming now to the energy of matter, for this we can also obtain a definite absolute value. But the energy of a body at rest is not equal to zero as might probably be imagined, following the analogy of the electromagnetic neutral field. The energy of a body at rest is equal to its mass multiplied by the square of the velocity of light. This is the so-called rest energy of the body, and is caused by its mechanical constitution and its temperature. If the body be set in motion by some force this energy value, which is of an enormous amount, does not make itself felt because the phenomenon of motion here arises from only a differentiation of energy. Such a conception could never have arisen from the energy principle itself. As a matter of fact it arises from the special theory of relativity, and it is a remarkable coincidence that it is just the theory of relativity which has led to the determination of an absolute value for the energy of a

physical system. This apparent paradox is explained by the simple fact that in the relativity theory there is the question of dependence on the reference system selected, whereas here there is the question of dependence on the physical state of the body under observation.

"Doesn't it in reality sound quite nonsensical to say that the energy of an atom of oxygen is sixteen times greater than that of an atom of hydrogen?" the purist might ask. We might answer that there would be no sense in such a statement if we could not speak of the hypothetical transformation of oxygen into hydrogen without involving a logical contradiction in the thought itself. But the idea of oxygen being one day changed into hydrogen does not involve any logical contradiction. Now, it is a mistake in these matters to speak of something as nonsensical unless it can be shown to be logically incoherent; and it would therefore seem more advisable to wait and see whether a day may not come when the problem of this transformation of oxygen into hydrogen may assume a reasonable significance. There are already signs that this time is at hand.

As in the case of electromagnetic and kinetic energy so too in all departments of physics, mechanics as well as electrodynamics, the movement has been away from dealing with differentials of energy toward dealing with absolute values of it. And this direction has invariably led to important results. When considering the phenomenon of heat radiation, for instance, it was always the strict rule to deal only with the difference between the radiation absorbed and that emitted; because all the heat rays that a body absorbs it can also give out. But in the theory of Prévost these two processes were sepnition of entropy put forward by Rudolf Clausius, if we are to measure the entropy of a body there must be a reversible process of some kind to enable us to determine the difference of entropy between the initial state and the final state of the process. In the light of this theory the concept of entropy originally referred not to a state but rather to a change of state, exactly as was the case in regard to atomic weight and energy. Indeed the earlier scientific notion was that the concept of entropy had a physical significance only where there could be a reversible process. It did not take long, however, before a broader concept was put forward and entropy began to be looked upon as a characteristic or inherent quality in the state of a body here and now. In this new way of looking at the case, however, there still remained an undefined additive constant, because one could still measure only the difference of entropy. Were we to follow the lead suggested by the Einstein experiments, and base the concept of entropy on the statistical laws governing the oscillations of a physical picture in relation to its thermodynamic state of equilibrium, even then we should only arrive at a measurement of differences, involved in a change of entropy, but never at the absolute value of entropy itself.

Is there then any way whereby we can hope to find an absolute value for entropy as has been found for energy? I do not think that the question can be answered on the basis of an analogy between these two cases. When such suggestions come to the fore I am always inclined to take my stand with the purists, who hold that it is senseless to try to arrive at the values of both *termini* from the value of the difference. If we are to keep our outlook clear we must always be very careful as to what can or cannot be deduced from a definition. In this regard the criterion of the purists is indispensable. We must do them the honor of saying that they are the conscientious wardens of order and purity in scientific methods. There is nothing more seductively dangerous in scientific work than the introduction of extraneous analogies into the problem at issue. That is a warning which needs to be sounded today even more insistently than before. But at the same time we must bear in mind the fact that physics is not a deductive science, and that its body of first principles is by no means fixed and unalterable. If a new axiom be suggested which we might introduce, then instead of rejecting it at once it ought to be put into quarantine, as one might say, and examined on its own merits for a clean bill of health. That clean bill of health which will give it a right to citizenship in physical science must be drawn up entirely free from prejudice as to the alien status of the axiom. The claim of the axiom must be adjudicated on the grounds of its ability to serve the cause of science in some direction where service is needed, and where the native axioms are unable to discharge such service. Once the new axiom has shown that it can solve hitherto insoluble problems, or at least produce a working hypothesis for their explanations, then it has a perfect right to be admitted.

Before indicating a definite line along which the question I have given above may eventually be answered, I will call attention to the difference between reversible and irreversible processes, and from this we shall understand the Boltzmann hypothesis which would suggest the measure by which the energy of a body at rest may be formulated. Therefore it is clear that the term "relativity" does not refer to physics as a whole and must not be taken out of its special scientific context. It would be quite superficial to take the relativity of time and space, and halt firmly within the confines of that concept without asking whither it leads. As a matter of fact, the concept of relativity is based on a more fundamental absolute than the erroneously assumed absolute which it has supplanted. Over and over again in the history of science it has happened that concepts which at one time were looked upon as absolute were subsequently shown to be only of relative value; and this is exactly what has happened in regard to the former concept of space and time. But when an absolute concept is thus relativized, this does not mean that the quest of the absolute becomes eliminated from scientific progress. It rather means that a more fundamental concept takes its place and a more fundamental advance is thus achieved. If we admit the concept of relativity at all we must admit the acceptance of an absolute, because it is out of this that the relative concept as such arises. Supposing, for instance, a scientific researcher worked for years and years on the problem of discovering the cause of some special event in nature and found all his efforts baffled, would he thereby be justified in declaring that the event has no cause at all? The fact is that we cannot relativize everything any more than we can define and explain everything. There are fundamentals that cannot be defined or explained, because they form the bedrock of all our knowledge. Every definition must necessarily rest on some concept which does not call for definition at all. And it is the

same with every form of proof. We cannot define a thing except in terms that are already known and accepted, and we cannot prove anything except from something that is already admitted. If we wish to establish a truth by the inductive method it must be on the basis of accepted facts. And if we wish to establish a truth by the process of deductive reasoning the principle from which the deduction proceeds must be accepted as absolute. Therefore the relativist concept must necessarily have the concept of the absolute as its foundation. If we once remove the absolute, then the whole relativist theory will fall to the ground, just as an overcoat would fall if the peg on which it hangs should disappear. These considerations are quite sufficient, I think, to suggest the reply which might be given to the counter argument of my imaginary disputant.

If eventually it should turn out possible to refer the atomic weights of all elements to the atomic weight of hydrogen, then we should have achieved one of the most fundamental results in the history of the scientific investigation of matter. The significance of it would be that in the light of this explanation matter could be proved to have one simple origin. Then the two factors of the hydrogen atom, namely, the positively charged hydrogen nucleus (the so-called proton) and the negatively charged electron, together with the elemental quantum of action, would represent the foundation stones on which the structure of the physical world is built. Now these quantities should be considered as absolute as long as they do not depend upon one another or something outside of them. There we should have the absolute once again, only at a higher level and in a simpler form. If we like world. If we should say, as several epistemologists do, that the absolute is to be found only in the individual's sensory data of perception, then there ought to be as many kinds of physical science as there are physicists, and we should be utterly unable to explain how it is that up to now each discoverer in physical science has been standing on the shoulders of his predecessors, as it were, and has taken their findings as the basis of his work. Indeed it is exclusively on the basis of co-operative labor and the acceptance by others of the findings of the various individual researchers, that we can explain the structure of physical science as we have it today. That we do not construct the external world to suit our own ends in the pursuit of science, but that vice versa the external world forces itself upon our recognition with its own elemental power, is a point which ought to be categorically asserted again and again in these positivistic times. From the fact that in studying the happenings of nature we strive to eliminate the contingent and accidental and to come finally to what is essential and necessary, it is clear that we always look for the basic thing behind the dependent thing, for what is absolute behind what is relative, for the reality behind the appearance and for what abides behind what is transitory. In my opinion, this is characteristic not only of physical science but of all science. Further, it is not merely a characteristic of all kinds of human endeavor to attain to the knowledge of any subject, but it is also characteristic of those branches of human effort that strive to formulate ideas of the good and the beautiful.

Here I am going wide of my purpose; for the plan I had in mind at the beginning of this essay was not to

The most striking instance of this is found in the practical application of physics. Not even the most confirmed skeptic can deny that we see and hear at a greater distance and command greater forces and speeds than an earlier generation; while it is equally certain that this progress is an enduring increase of knowledge, which is in no danger of being described as an error and rejected at any future date.

Secondly, it is a very striking fact that the impulse toward simplification and improvement of the world picture of physics was due in each instance to some kind of novel observation—that is, to some event in the world of sense. But at the same moment the structure of this physical world consistently moved farther and farther away from the world of sense and lost its former anthropomorphic character. Still further, physical sensations have been progressively eliminated, as for example in physical optics, in which the human eye no longer plays any part at all. Thus the physical world has become progressively more and more abstract; purely formal mathematical operations play a growing part, while qualitative differences tend to be explained more and more by means of quantitative differences.

Now we have already pointed out that the physical view of the world has been continually perfected and also related to the world of sense. If this fact is added to those mentioned in the last paragraph, the result is extraordinarily striking; at first, indeed, it appears completely paradoxical. Of this apparent paradox there is, in my opinion, only one rational explanation. This consists in saying that as the view of the physical world is perfected, it simultaneously recedes from the world of sense; and this process is tantamount to an approach to the world of reality. I have no logical proof on which to base this opinion; it is impossible to demonstrate the existence of the real world by purely rational methods: but at the same time it is equally impossible ever to refute it by logical methods. The final decision must rest upon a common-sense view of the world, and the old maxim still remains true that that world view is the best which is the most fruitful. Physics would occupy an exceptional position among all the other sciences if it did not recognize the rule that the most far-reaching and valuable results of investigation can only be obtained by following a road leading to a goal which is theoretically unobtainable. This goal is the apprehension of true reality.

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What changes have taken place in the physical view of the world during the last twenty years? We all know that the changes which have occurred during this period are among the most profound that have ever risen in the evolution of any science; we also know that the process of change has not yet come to an end. Nevertheless, it would appear that in this flux of change certain characteristic forms of the structure of this new world are beginning to crystallize; and it is certainly worth while to attempt a description of these forms, if only in order to suggest certain improvements.

If we compare the old theory with the new, we find that the process of tracing back all qualitative distinctions to quantitative distinctions has been advanced very considerably. All the various chemical phenomena, for example, have now been explained by numerical and spatial relations. According to the modern view there are no more than two ultimate substances, namely, positive and negative electricity. Each of these cona certain amount of confusion into the traditional ideas of time and space; in the long run, however, it has proved to be the completion and culmination of the structure of classical physics. To express the positive results of the special theory of relativity in a single word, it might be described as the fusion of time and space in one unitary concept. It is not, of course, asserted that time and space are absolutely similar in nature; their relation resembles that between a real number and an imaginary number, when these are combined together to form the unified concept of a complex number. Looked at in this way, Einstein's work for physics closely resembles that of Gauss for mathematics. We might further continue the comparison by saying that the transition from the special to the general theory of relativity is the counterpart in physics to the transition from linear functions to the general theory of functions in mathematics.

Few comparisons are entirely exact, and the present is no exception to the rule. At the same time it gives a good idea of the fact that the introduction of the theory of relativity into the physical view of the world is one of the most important steps toward conferring unity and completeness. This appears clearly in the results of the theory of relativity, especially in the fusing of momentum and energy, in the identification of the concept of mass with the concept of energy, of inertial with ponderable mass, and in the reduction of the laws of gravitation to Riemann's geometry.

Brief though these main outlines are, they contain a vast mass of new knowledge. The new ideas mentioned apply to all natural events great and small, beginning

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The idea of the universe as thus far described appeared almost perfectly adapted to its purpose; but this state of affairs has suddenly been upset by the quantum theory. Here again I shall attempt to describe the characteristic idea of this hypothesis in one word. We may say, then, that its essence consists in the fact that it introduces a new and universal constant, namely the elementary quantum of action. It was this constant which, like a new and mysterious messenger from the real world, insisted on turning up in every kind of measurement, and continued to claim a place for itself. On the other hand, it seemed so incompatible with the traditional view of the universe provided by physics that it eventually destroyed the framework of this older view.

For a time it seemed that a complete collapse of classical physics was not beyond the bounds of possibility; gradually, however, it appeared, as had been confidently expected by all who believed in the steady

The question may now be asked whether modern physics differs at all from the older physics, if all these foundations of classical physics have remained untouched. It is easy to find an answer to this question by examining the elementary quantum of action somewhat more closely. It implies that in principle an equation can be established between energy and frequency; $E = h\nu$.* It is this equation which classical physics utterly fails to explain. The fact itself is so baffling because energy and frequency possess different dimensions; energy is a dynamic magnitude, whereas frequency is a kinematic magnitude. This fact in itself, however, does not contain a contradiction. The quantum theory postulates a direct connection between dynamics and kinematics; this connection is due to the fact that the unit of energy, and consequently the unit of mass, are based upon the units of length and of time; thus the connection, so far from being a contradiction, enriches and rounds off the classical theory. There is, nevertheless, a direct contradiction, which renders the new theory incompatible with the classical theory. The following considerations make clear this contradiction. Frequency is a local magnitude, and has a definite meaning only for a certain point in space; this is true alike of mechanical, electric, and magnetic vibrations, so that all that is requisite is to observe the point in question for a sufficient time. Energy, on the other hand, is an additive

* In this equation E stands for energy, and ν for frequency, that is, the number of vibrations per second. For example, light vibrations range from about 400 million million per second to about 800 million million. h represents Planck's constant, discovered by the author of this work. It is an unchanging or invariable quantity, and extremely minute, its value being 655 preceded by 26 decimal places. [TRANS.]

quantity; so that according to the classical theory it is meaningless to speak of energy at a certain point, since it is essential to state the physical system the energy of which is under discussion; just as it is similarly impossible to speak of a definite velocity unless the system be indicated to which velocity is referred. Now we are at liberty to choose whatever physical system we please, either little or great; and consequently the value of the energy is always to a certain extent arbitrary. The difficulty, then, consists in the fact that this arbitrary energy is supposed to be equated with a localized frequency. The gulf between these two concepts should now be clearly apparent: and in order to bridge this gulf a step of fundamental importance must be taken. This step does imply a break with those assumptions which classical physics has always regarded and employed as axiomatic.

Hitherto it had been believed that the only kind of causality with which any system of physics could operate was one in which all the events of the physical world—by which, as usual, I mean not the real world but the world view of physics—might be explained as being composed of local events taking place in a number of individual and infinitely small parts of space. It was further believed that each of these elementary events was completely determined by a set of laws without respect to the other events; and was determined exclusively by the local events in its immediate temporal and spatial vicinity. Let us take a concrete instance of sufficiently general application. We will assume that the physical system under consideration consists of a system of particles, moving in a conservative field of force of constant the same part as do the equations established by Newton, Lagrange, and Hamilton in classical mechanics. Nevertheless, there is an important distinction between these equations, consisting in the fact that in the latter equations the co-ordinates of the configuration point are not functions of time, but independent variables. Accordingly, while for any given system the classical equations of motion were more or less numerous and corresponded to the number of degrees of freedom of the system, there can be only one single quantum equation for each system. In course of time the configuration point of classical theory describes a definite curve; on the other hand, the configuration point of the material wave fills at any given time the whole of infinite space, including those parts of space where potential energy is greater than the total energy, so that according to the classical theory, kinetic energy would become negative in these parts of space, and the momentum imaginary. This case resembles the so-called total reflection of light, where according to geometrical optics light is completely reflected, because the angle of refraction becomes imaginary; whereas according to the wave theory of light, it is perfectly possible for light to penetrate into the second medium, even if it cannot do so as a plane wave.

At the same time, the fact that there are points in configuration space where the potential energy exceeds the total energy is of extreme importance for quantum mechanics. Calculation shows that in every such instance a finite wave corresponds not to any given value of the energy constant, but corresponds only to certain definite values: the so-called characteristic energy values, which can be calculated from the wave equation and have difat least approximately, if we make use of the wave within a certain narrow range of frequency.

According to wave mechanics, both the position and the momentum of a system of particles can never be defined without some uncertainty. Now the fact is that between these two kinds of uncertainty there is a definite relation. This follows from the simple reflection that if the waves of which we make use are to cancel each other through interference outside the above-mentioned small configuration region, then in spite of their small differences in frequency, noticeable differences in propagation must appear at the opposite boundaries of the region. If in accordance with the quantum postulate, we substitute differences of momentum for differences of propagation, we obtain Heisenberg's principle, which states that the product of the uncertainty of position and uncertainty of momentum is at least of the same order of magnitude as the quantum of action.]

THE more accurately the position of the configuration point is ascertained, the less accurate is the amount of momentum; and conversely. These two kinds of uncertainty are thus in a certain sense complementary; this complementariness is limited by the fact that momentum can under certain conditions be defined with absolute accuracy in wave mechanics, whereas the position of a configuration point always remains uncertain within a finite region.

Now this relation of uncertainty, established by Heisenberg, is something quite unheard of in classical mechanics. It had always been known, of course, that every measurement is subject to a certain amount of inaccuracy; but it had always been assumed that an improvement in method would lead to an improvement in accuracy, and that this process could be carried on indefinitely. According to Heisenberg, however, there is a definite limit to the accuracy obtainable. What is most curious is that this limit does not affect position and velocity separately, but only the two when combined together. In principle, either taken by itself can be measured with absolute accuracy, but only at the cost of the accuracy of the other.

Strange as this assertion may seem, it is definitely established by a variety of facts. I will give one example to illustrate this. The most direct and accurate means of ascertaining the position of a particle consists in the optical method, when the particle is looked at with the naked eye or through a microscope, or else is photographed. Now for this purpose the particle in question must be illuminated. If this is done the definition becomes more accurate; consequently the measurement becomes more exact in proportion as the light waves employed become shorter and shorter. In this sense, then, any desired degree of accuracy can be attained. On the other hand there is also a disadvantage, which affects the measurement of velocity. Where the masses in question have a certain magnitude, the effect of light upon the illuminated object may be disregarded. But the case is altered if a very small mass, e.g., a single electron, is selected; because each ray of light, which strikes the electron and is reflected by it, gives it a distinct impulse; and the shorter the light wave the more powerful is this impulse. Consequently, the shorter the light wave the more accurately is it possible to determine position;

the framework of atomic physics; consequently the methods employed will always be extremely delicate. At present we can only say that hitherto no fact has been discovered which throws doubt on the applicability in physics of all these conclusions.

The fact is that since the wave equation was first formulated, the theory has been developing at a most remarkable rate. It is impossible within the framework of a small volume to mention all the extensions and applications of the theory which have been evolved within recent years. I shall confine myself to the so-called stress of protons and electrons; the formulation of quantum mechanics in terms of relativity; the application of the theory to molecular problems, and the treatment of the so-called "many-body problem," i.e., its application to a system containing a number of exactly similar particles. Here statistical questions, relating to the number of possible states within a system, having a given energy, are particularly important; they also have a bearing on the calculation of the entropy of the system.

Finally, I cannot here enter in detail upon the physics of light quanta. In a certain sense this study has developed in the opposite direction from the physics of particles. Originally Maxwell's theory of electromagnetic waves dominated this region, and it was not seen until later that we must assume the existence of discrete light particles; in other words that the electromagnetic waves, like the material waves, must be interpreted as waves of probability.

Perhaps there is no more impressive proof of the fact that a pure wave theory cannot satisfy the demands of modern physics any more than a pure corpuscular theory. Both theories, in fact, represent extreme limiting cases. The corpuscular theory, which is the basis of classical mechanics, does justice to the configuration of a system, but fails to determine the values of its energy and of momentum; conversely the wave theory, which is characteristic of classical electrodynamics, can give an account of energy and momentum, but excludes the idea of the localization of light particles. The standard case is represented by the intermediate region, where both theories play equally important parts; this region can be approached from either side, although at present a close approach is impossible. Here many obscure points await solution, and it remains to be seen which of the various methods employed for their solution best leads to the goal. Among them we may mention the matrix calculus invented by Heisenberg, Born, and Jordan, the wave theory due to De Broglie and Schrödinger, and the mathematics of the q numbers introduced by Dirac.

this peaceful international collaboration. It is in this reciprocal action of experiment and theory—which is at once a stimulus to and a check upon progress—that we see the surest and indeed the only guarantee of the future advance of physics.

What will be the ultimate goal? I had occasion at the beginning to point out that research in general has a twofold aim—the effective domination of the world of sense, and the complete understanding of the real world; and that both these aims are in principle unattainable. But it would be a mistake to be discouraged on this account. Both our theoretical and practical tangible results are too great to warrant discouragement; and every day adds to them. Indeed, there is perhaps some justification for seeing in the very fact that this goal is unattainable, and the struggle unending, a blessing for the human mind in its search after knowledge. For it is in this way that its two noblest impulses—enthusiasm and reverence—are preserved and inspired anew.

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What now do we mean by physical law? A physical law is any proposition enunciating a fixed and absolutely valid connection between measurable physical quantities—a connection which permits us to calculate one of these quantities if the others have been discovered by measurement. The highest and most keenly desired aim of any physicist is to obtain the most perfect possible knowledge of the laws of physics, whether he looks at them from a utilitarian point of view and values them because they enable him to save himself the trouble of costly measurements, or takes a deeper view and looks to them for satisfaction of a profound yearning after knowledge and for a firm basis of natural science.

How do we discover the individual laws of physics, and what is their nature? It should be remarked, to begin with, that we have no right to assume that any physical laws exist, or if they have existed up to now, that they will continue to exist in a similar manner in future. It is perfectly conceivable that one fine day nature should cause an unexpected event to occur which would baffle us all; and if this were to happen we would be powerless to make any objection, even if the result would be that, in spite of our endeavors, we should fail to introduce order into the resulting confusion. In such an event, the only course open to science would be to declare itself bankrupt. For this reason, science is compelled to begin by the general assumption that a general rule of law dominates throughout nature, or, in Kantian terminology, to treat the concept of causality as being one of the categories which are given a priori and without which no kind of knowledge can be attained.

From this it follows that the nature of the laws of physics, and the content of these laws, cannot be obtained by pure thought; the only possible method is to turn to the investigation of nature, to collect the greatest possible mass of varied experiences, to compare these and to generalize them in the simplest and most comprehensive proposition. In other words, we must have recourse to the method of induction.

The content of an experience is proportionally richer as the measurements upon which it is based are more exact. Hence it is obvious that the advance of physical knowledge is closely bound up with the accuracy of physical instruments and with the technique of measurement. The latest developments of physics provide us with striking examples of the truth of this. Measurement alone, however, does not suffice. For each measurement is an individual event standing by itself; as such, it is determined by special circumstances, especially by a definite place and a definite time, but also by a definite measuring instrument, and by a definite observer. It far greater scope than that of force; it reaches beyond the sphere of mechanics into that of chemical affinities, where we are no longer concerned with Newtonian force. It must be admitted that the idea of potential has not the advantage of immediate obviousness which belongs to force by virtue of its anthropomorphic quality; whence it follows that the elimination of the concept of force renders the laws of physics much less obvious and easy of understanding. Yet this development is quite natural; the laws of physics have no consideration for the human senses; they depend upon facts, and not upon the obviousness of facts.

In my opinion, the teaching of mechanics will still have to begin with Newtonian force, just as optics begins with the sensation of color, and thermodynamics with the sensation of warmth, despite the fact that a more precise basis is substituted later on. Again, it must not be forgotten that the significance of all physical concepts and propositions ultimately does depend on their relation to the human senses. This is indeed characteristic of the peculiar methods employed in physical research. If we wish to form concepts and hypotheses applicable to physics, we must begin by having recourse to our powers of imagination; and these depend upon our specific sensations, which are the only source of all our ideas. But to obtain physical laws we must abstract exhaustively from the images introduced, and remove from the definitions set up all irrelevant elements and all imagery which do not stand in a logical connection with the measurements obtained. Once we have formulated physical laws, and reached definite conclusions by mathematical processes, the results which we have obtained quantity containing a relatively small number of molecules, then the average of their velocities will vary; and the variation will be the greater, the smaller is the quantity of liquid. This principle can nowadays be regarded as a fact fully proved by experiment. One of the most striking illustrations is what is known as the Brownian movement, which can be observed through the microscope in small particles of powder suspended in liquid. These particles are driven backward and forward by the invisible molecules of the liquid; the movement is the more pronounced the higher is the temperature. If we make the further assumption, to which in principle there is no objection, that each individual impulse is a reversible event governed by the strict elementary laws of dynamics, then we may say that the introduction of a microscopic method of examination shows that the laws governing the irreversible processes, or what is the same thing, the laws based upon statistics and mere rough approximation, can be traced back to dynamic, accurate, and absolute laws.

The striking results reached by the introduction of statistical laws in many branches of physical research in recent times have produced a remarkable change in the views of physicists. They no longer, as in the earlier days of energetics, deny or attempt to cast doubt upon the existence of irreversible processes; instead, the attempt is frequently made to place statistical laws in the foreground, and to subordinate to them laws hitherto regarded as dynamic, including even the law of gravitation. In other words, an attempt is made to exclude absolute law from nature. And indeed, we cannot but be struck by the fact that the natural phenomena which we

can investigate and measure can never be expressed by absolutely accurate numbers; for they inevitably contain a certain inaccuracy introduced by the unavoidable defects of measurement itself. Hence it follows that we shall never succeed in determining by measurement whether a natural law is absolutely valid. If we consider the question from the standpoint of the theory of knowledge we come to the same conclusion. For if we cannot even prove that nature is governed by law (a difficulty which we meet with at the very outset) a fortiori we shall be unable to demonstrate that such law is absolute.

Hence from a logical point of view, we must admit every justification for the hypothesis that the only kind of law in nature is statistical. It is a different question whether this assumption is expedient in physical research; and I feel strongly inclined to answer this question in the negative. We must consider in the first instance that the only type of law fully satisfying our desire for knowledge is the strictly dynamic type, while every statistical law is fundamentally unsatisfactory, for the simple reason that it has no absolute validity but admits of exceptions in certain cases; so that we are continually faced by the question what these particular exceptional cases are.

Questions of this nature constitute the strongest argument in favor of the extension and further refinement of experimental methods. If it is assumed that statistical laws are the ultimate and most profound type in existence, then there is no reason in theory why, when dealing with any particular statistical law, we should ask: what are the causes of the variations in the phenomena? Actually, however, the most important advances in the accidental variations of the climatological curves, of population statistics and mortality tables, are in each instance subject to strict causality; similarly, physicists will always admit that such questions are strictly relevant as that which asks why one of two neighboring atoms of uranium exploded many millions of years before the other.

All studies dealing with the behavior of the human mind are equally compelled to assume the existence of strict causality. The opponents of this view have frequently brought forward against it the existence of free will. In fact, however, there is no contradiction here; human free will is perfectly compatible with the universal rule of strict causality—a view which I have had occasion to demonstrate in detail elsewhere. But as my arguments on this subject have been seriously misunderstood in certain quarters, and since this subject is surely of considerable importance, I propose to discuss it briefly here.

The existence of strict causality implies that the actions, the mental processes, and especially the will of every individual are completely determined at any given moment by the state of his mind, taken as a whole, in the previous moment, and by any influences acting upon him coming from the external world. We have no reason whatever for doubting the truth of this assertion. But the question of free will is not concerned with the question whether there is such a definite connection, but whether the person in question is aware of this connection. This, and this alone, determines whether a person can or cannot feel free. If a man were able to forecast his own future solely on the ground of causality, then and then only we would have to deny this consciousness of freedom of the will. Such a contingency is, however, impossible, since it contains a logical contradiction. Complete knowledge implies that the object apprehended is not altered by any events taking place in the knowing subject; and if subject and object are identical, this assumption does not apply. To put it more concretely, the knowledge of any motive or of any activity of will is an inner experience, from which a fresh motive may spring; consequently such an awareness increases the number of possible motives. But as soon as this is recognized, the recognition brings about a fresh act of awareness, which in its turn can generate yet another activity of the will. In this way the chain proceeds, without it ever being possible to reach a motive which is definitely decisive for any future action; in other words, to reach an awareness which is not in its turn the occasion of a fresh act of will. When we look back upon a finished action, which we can contemplate as a whole, the case is completely different. Here knowledge no longer influences will, and hence a strictly causal consideration of motives and will is possible, at least in theory.

If these considerations appear unintelligible—if it is thought that a mind could completely grasp the causes of its present state, provided it were intelligent enough—then such an argument is akin to saying that a giant who is big enough to look down on everybody else should be able to look down on himself as well. But no person, however clever, can derive the decisive motives of his own conscious actions from causal law alone; he needs another law—ethical law, for which the highest intelligence and the most subtle self-analysis are no adequate substitute.

better than co-operate in extending the theory of relativity and in pushing its conclusions as far as possible, since this is the only means of refuting it through experience. Such an undertaking is the less difficult because the assertions made by the theory of relativity are simple and comparatively easy to apprehend, so that they fit into the framework of classical physics without any difficulty.

Indeed, if there were no historical objections I personally would not hesitate for a moment to include the theory of relativity within the body of classical physics. In a manner the theory of relativity is the crowning point of physics, since by merging the ideas of time and space it has also succeeded in uniting under a higher point of view such concepts as those of mass, energy, gravitation, and inertia. As the result of this novel view we have the perfectly symmetrical form which the laws of the conservation of energy and of momentum now assume; for these laws follow with equal validity from the principle of least action—that most comprehensive of all physical laws which governs equally mechanics and electrodynamics.

Now over against this strikingly imposing and harmonious structure there stands the quantum theory, an extraneous and threatening explosive body which has already succeeded in producing a wide and deep fissure throughout the whole of the structure. Unlike the theory of relativity, the quantum theory is not complete in itself. It is not a single, harmonious, and perfectly transparent idea, modifying the traditional facts and concepts of physics by means of a change which, though of the utmost significance in theory, is practically hardly noticeable. On the contrary, it first arose as a means of escape from an *impasse* reached by classical physics in one particular branch of its studies—the explanation of the laws of radiant heat. It was soon seen, however, that it also solved with ease, or at least considerably helped to elucidate, other problems which were causing unmistakable difficulties to the classical theory, such as photoelectric phenomena, specific heat, ionization, and chemical reactions. Thus it was quickly realized that the quantum theory must be regarded, not merely as a working hypothesis, but as a new and fundamental principle of physics, whose significance becomes evident wherever we are dealing with rapid and subtle phenomena.

Now here we are faced with a difficulty. This does not so much consist in the fact that the quantum theory contradicts the traditional views; if that were all, it follows from what has been said that the difficulty need not be taken very seriously. It arises from the fact that in the course of time it has become increasingly obvious that the quantum theory unequivocally denies certain fundamental views which are essential to the whole structure of the classical theory. Hence the introduction of the quantum theory is not a modification of the classical theory, as is the case with the theory of relativity: it is a complete break with the classical theory.

Now if the quantum theory were superior or equal to the classical theory at all points, it would be not only feasible but necessary to abandon the latter in favor of the former. This, however, is definitely not the case. For there are parts of physics, among them the wide region

of the phenomena of interference, where the classical theory has proved its validity in every detail, even when subjected to the most delicate measurements; while the quantum theory, at least in its present form, is in these respects completely useless. It is not the case that the quantum theory cannot be applied, but that, when applied, the results reached do not agree with experience.

The result of this state of affairs is that at the present moment each theory has what may be called its own preserve, where it is safe from attack, while there is also an intermediate region—e.g., that of the phenomena of the dispersion and scattering of light—where the two theories compete with varying fortunes. The two theories are approximately of equal usefulness, so that physicists are guided in the choice of theory by their private predilections—an uncomfortable, and in the long run, an intolerable state of affairs for anyone desirous of reaching the true facts.

To illustrate this curious condition of things I will select a particular example from a very large number collected by workers in the field of theory and of practice. I begin by stating two facts. Let us imagine two fine pencils of rays of violet light, produced by placing an opaque screen with two small holes over against the light which is given out from a point source. The two pencils of rays emerging from the holes can be reflected so that they meet on the surface of a white wall at some distance away. In this case the spot of light which they jointly produce on the wall is not uniformly bright, but is traversed by dark lines. This is the first fact. The second is this—if any metal that is sensitive to light is placed in the path of one of these rays, the metal will continually emit electrons with a velocity independent of the intensity of the light.

Now if the intensity of the source of light is allowed to decrease, then in the first case, according to all the results hitherto obtained, the dark lines remain quite unchanged; it is only the strength of the illumination that decreases. In the other case, however, the velocity of the electrons emitted also remains quite unchanged, and the only change that takes place is that the emission becomes less copious.

Now how do the theories account for these two facts? The first is adequately explained by the classical theory as follows: at every point of the white wall which is simultaneously illuminated by the two pencils of rays, the two rays which meet at this point either strengthen or else weaken each other, according to the relations between their respective wave lengths. The second fact is equally satisfactorily explained by the quantum theory, which maintains that the energy of the rays falls on the sensitive metal, not in a continuous flow, but in an intermittent succession of more or less numerous, equal and indivisible quanta, and that each quantum, as it impinges on the metal, detaches one electron from the mass. On the other hand, all attempts have failed hitherto to explain the lines of interference by the quantum theory and the photoelectric effect by the classical theory. For if the energy radiated really travels only in indivisible quanta, then a quantum emitted from the source of light can pass only through one or else the other of the two holes in the opaque screen; while if the light is sufficiently feeble, it is also impossible for two

distinct rays to impinge simultaneously on a single point on the white wall; hence interference becomes impossible. In fact the lines invariably disappear completely, as soon as one of the rays is cut off.

On the other hand, if the energy radiated from a point source of light spreads out uniformly through space, its intensity must necessarily be diminished. Now it is not easy to see how the velocity with which an electron is emitted from the sensitive metal can be equally great whether it is subjected to very powerful or to very weak radiation. Naturally many attempts have been made to get over this difficulty. Perhaps the most obvious way was to assume that the energy of the electron emitted by the metal is not derived from the radiation falling on it, but that it comes from the interior of the metal, so that the effect of the radiation is merely to set it free in the same way as a spark sets free the latent energy of gunpowder. It has, however, not proved possible to demonstrate that there is such a source of energy, or even to make it appear plausible that there should be such a source. Another supposition is that, while the energy of the electrons is derived from the radiation impinging upon them, the electrons themselves are not actually emitted from the metal until this has been subjected to the illumination for a time sufficiently long to allow the energy necessary for a definite velocity to have been accumulated. This process, however, might take minutes or even hours, whereas in fact the phenomenon repeatedly takes place very much sooner. Light is thrown on the profound importance of these difficulties by the fact that in highly influential quarters the suggestion has arisen of sacrificing the validity of the prinspeculations remain in the air so long as they are unsubstantiated by definite facts of experience; and we must hope and trust that the experimental skill of physicists, which in the past has so often definitely decided questions full of doubt and difficulty, will succeed in resolving the difficulties of the present obscure question. In any case there can be no doubt that the parts of the structure of classical physics, which have had to be discarded as valueless under the pressure of the quantum theory, will be supplanted by a sounder and more adequate structure.

To conclude: we have seen that the study of physics, which a generation ago was one of the oldest and most mature of natural sciences, has today entered upon a period of storm and stress which promises to be the most interesting of all. There can be little doubt that in passing through this period we shall be led, not only to the discovery of new natural phenomena but also to new insight into the secrets of the theory of knowledge. It may be that in the latter field many surprises await us, and that certain views, eclipsed at the moment, may revive and acquire a new significance. For this reason a careful study of the views and ideas of our great philosophers might prove extremely valuable in this direction.

There have been times when science and philosophy were alien, if not actually antagonistic to each other. These times have passed. Philosophers have realized that they have no right to dictate to scientists their aims and the methods for attaining them; and scientists have learned that the starting point of their investigations does not lie solely in the perceptions of the senses, and that science cannot exist without some small portion of metaphysics. Modern physics impresses us particularly with the truth of the old doctrine which teaches that there are realities existing apart from our sense perceptions, and that there are problems and conflicts where these realities are of greater value for us than the richest treasures of the world of experience.

It is important at this point to state that there is no one definite principle available a priori and enabling a classification suitable for every purpose to be made. This applies equally to every science. Hence it is impossible in this connection to assert that any science possesses a structure evolving from its own nature inevitably and apart from any arbitrary presupposition. It is important that this fact should be clearly grasped: it is of a fundamental significance because it demonstrates that it is essential, if there is to be any scientific knowledge, to determine the principle in accordance with which its studies are to be pursued. This determination cannot be made merely in accordance with practical considerations; questions of value also play their part.

Let us take a simple example from the most mature and exact of all sciences, mathematics. Mathematics deals with the magnitude of numbers. In order to obtain a survey of all numbers the obvious method would be to classify them by magnitude; in which case any two numbers are close to each other in proportion as the difference between them is small. Let us take two numbers which are practically equal in magnitude, one of them being the square root of 2 and the other 1.41421356237. The former figure is a few billionths greater than the latter and in every numerical calculation in physics or in astronomy the two numbers can be treated as completely identical. So soon, however, as numbers are classified in accordance with their origin and not in accordance with their magnitude a fundamental difference between the two numbers arises. The decimal fraction is a rational number and can be expressed by the ratio between two integers, while the square root is irraof an individual electron at any given time; what it does is to state the probability that an electron will be at a given place at a given time; or alternatively, given a multitude of electrons, it states the number which in any given time will be at a given place.

This is a law of a purely statistical character. The fact that it has been confirmed by all measurements hitherto made, and the further fact that there is such a thing as the uncertainty relation, has induced certain physicists to conclude that statistical laws are the only valid foundations of every physical law, more particularly in the field of atomic physics; and to declare that any question about the causality of individual events is, physically, meaningless.

We here reach a point whose discussion is of particular importance, since it leads us to a fundamental question: what is the task and what are the achievements of physics? If we hold that the object of physics is to discover the laws governing the relation between the real events of nature, then causality becomes a part of physics, and its deliberate elimination must give rise to certain misgivings.

It should first be observed that the validity of statistical laws is entirely compatible with a strict causality. Classical physics contains numerous examples. Thus, we may explain the pressure of a gas on the wall of the containing vessel as due to the irregular impingement of numerous gas molecules flying about in all directions; but this explanation is compatible with the admission that the impingement of any one molecule upon another or upon the wall is governed by law and hence is completely determined causally. It may be objected that a

strict causality can be regarded as definitely proved only if we are in a position to predict the entire course of the event; and it might be added that nobody can check the movement of any single molecule. To this we might reply that a rigorously exact prediction is never possible of any natural event, so that the validity of the law of causality can never be demonstrated by an immediate and exact experiment, since every measurement, however exact, inevitably involves certain errors of observation. Yet in spite of this the result of the measurement as well as individual errors of observation are attributed to definite causes. When we watch the waves breaking on the seashore, we have every right to feel convinced that the movement of every bubble is due to strict causal law, although we could never hope to follow its rise and fall, still less to calculate it in advance.

It is at this point that the uncertainty relation is brought forward. While classical physics was fashionable, it might be hoped that the inevitable errors of observation could be reduced beneath any given limit by an appropriate increase in the accuracy of measurements. This hope was destroyed by the discovery of Planck's constant, since the latter implies a fixed objective limitation of the exactitude which can be reached, within which limit there is no causality but only doubt and contingency.

We have already prepared a reply to this objection. The reason why the measurements of atomic physics are inexact need not necessarily be looked for in any failure of causality; it may equally well consist in the formulation of faulty concepts and hence of inappropriate questions.

It is precisely the reciprocal influence between the measurement and the real event which enabled us to understand the uncertainty relation at least to a certain degree. According to this view we can no more follow the movement of the individual electron than we can see a colored picture whose dimensions are smaller than the wave length of its color.

It is true that we must reject as meaningless the hope that it might eventually prove possible indefinitely to reduce the inaccuracy of physical measurements by improving the instrument. Yet the existence of an objective limit like Planck's constant is a sure indication that a certain novel law is at work which has certainly nothing to do with statistics. Like Planck's constant, every other elementary constant, e.g., the charge or mass of an electron, is a definite real magnitude; and it seems wholly absurd to attribute a certain fundamental inexactitude to these universal constants, as those who deny causality would have to do if they wish to remain consistent.

The fact that there is a limit to the accuracy of the measurements in atomic physics becomes further intelligible if we consider that the instruments themselves consist of atoms and that the accuracy of any measuring instrument is limited by its own sensitiveness. A weighbridge cannot weigh to the nearest milligram.

Now what can we do if the best that we have is a weighbridge and there is no hope of obtaining anything more accurate? Would it not be better to give up hope of obtaining exact weights and to declare the pursuit of the milligram to be meaningless, rather than to pursue a task which cannot be solved by direct measurement? This argument underestimates the importance of theory: for theory takes us beyond direct measurement in a way which cannot be foretold a priori, and it does so by means of the so-called intellectual experiments which render us largely independent of the defects of the actual instruments.

It is wholly absurd to maintain that an intellectual experiment is important only in proportion as it can be checked by measurement; for if this were so, there could be no exact geometrical proof. A line drawn on paper is not really a line but a more or less narrow strip, and a point a larger or smaller spot. Yet nobody doubts that geometrical constructions yield a rigorous proof.

The intellectual experiment carries the mind of the investigator beyond the world and beyond actual measuring instruments and enables him to form hypotheses and to formulate questions which, when checked by actual experiment, enable him to perceive new laws even when these do not admit of direct measurement. An intellectual experiment is not tied down to any limits of accuracy, for thoughts are more subtle than atoms or electrons, nor is there any danger that the event which is measured can be influenced by the measuring instrument. An intellectual experiment requires one condition only for its success, and this is the admission of the validity of any non-self-contradictory law governing the relations between the events under observation. We cannot hope to find what is assumed not to be existent.

Admittedly an intellectual experiment is an abstraction; an abstraction, however, as essential to the experimenter and to the theorist as the abstract assumption that there is a real external world. Whenever we observe an event taking place in nature we must assume that something is happening independently of the observer, and conversely we must endeavor to eliminate as far as possible the defects of our senses and of our methods of measurement in order to grasp the details of the event with greater perfection. There is a kind of opposition between these two abstractions: while the real external world is the object, the ideal spirit which contemplates it is the subject. Neither can be logically demonstrated and hence no *reductio ad absurdum* is possible if their existence is denied. The history of physics bears witness, however, that they have played a decisive part throughout its development. The choicest and most original minds, men like Kepler, Newton, Leibniz, and Faraday, were inspired by the belief in the reality of the external world and in the rule of a higher reason in and beyond it.

It should never be forgotten that the most vital ideas in physics have this twofold origin. In the first instance the form which these ideas take is due to the peculiar imagination of the individual scientist: in course of time, however, they assume a more definite and independent form. It is true that there have always been in physics a number of erroneous ideas on which a quantity of labor was wasted: yet on the other hand, many problems which were at first rejected as meaningless by keen critics were eventually seen to possess the highest significance. Fifty years ago positivist physicists considered it meaningless to ask after the determination of the weight of a single atom—an illusory problem not admitting scientific treatment. Today the weight of an atom can be stated to within its ten-thousandth part, although our most delicate scales are no more fit to weigh

it than a weighbridge is to determine milligrams. One should therefore beware of declaring meaningless a problem whose solution is not immediately apparent; there is no criterion for deciding a priori whether any given problem in physics has a meaning or not, a point frequently overlooked by the positivists. The only means of judging a problem correctly consists in examining the conclusions to which it leads. Now the assumption that there are rigid laws applicable to physics is of such fundamental importance that we should hesitate before we declare the question whether such laws are applicable to atomic physics to be a meaningless one. Our first endeavor, on the contrary, should be to trace out the problem of the applicability of laws in this field.

Our first step should be to ask why classical physics fails in the question of causality when the interference arising from the measuring instrument and the inadequate accuracy of the latter are both insufficient to explain this failure. Plainly we are forced to adopt the obvious but radical assumption that the elementary concepts of classical physics cease to be applicable in atomic physics.

Classical physics is based on the assumption that its laws are most clearly revealed in the infinitely small; for it assumes that the course of a physical event anywhere in the universe is completely determined by the state prevailing at this place and its immediate vicinity. Hence such physical magnitudes relating to the state of the physical event as position, velocity, intensity of the electric and magnetic field, etc., are of a purely local character, and the laws governing their relation can be completely expressed by spatial-temporal differential equations between these magnitudes. Clearly, however, this will not suffice for atomic physics, so that the above concepts must be made more complete or more universal. In which direction, however, is this to be done? Some indication may perhaps be found in the recognition, which is daily spreading wider, that the spatial-temporal differential equations do not suffice to exhaust the content of the events within a physical system and that the liminal conditions must also be taken into consideration. This applies even to wave mechanics. Now the field of the liminal conditions is always finite and its immediate interference in the causal nexus is a new manner of looking at causality and one hitherto foreign to classical physics.

The future will show whether progress is possible in this direction and how far it will lead. But whatever results it may ultimately reveal, it is certain that it will never enable us to grasp the real world in its totality any more than human intelligence will ever rise into the sphere of ideal spirit: these will always remain abstractions which by their very definition lie outside actuality. Nothing, however, forbids us to believe that we can progress steadily and without interruption to this unattainable goal; and it is precisely the task of science with its continual self-correction and self-improvement to work in this direction without cease once it has been recognized that it is a hopeful direction. This progress will be a real one and not an aimless zigzag, as is proved by the fact that each new stage reached enables us to survey all the previous stages, while those which remain to be covered are still obscure; just as a climber trying to reach higher altitudes looks down upon the distance

he has covered in order to gain knowledge for the further ascent. A scientist is happy, not in resting on his attainments but in the steady acquisition of fresh knowledge.

I have so far confined myself to physics; but it may be felt that what has been said has a wider application. Natural science and the intellectual sciences cannot be rigorously separated. They form a single interconnected system, and if they are touched at any part the effects are felt through all the ramifications of the whole, the totality of which is forthwith set in motion. It would be absurd to assume that a fixed and certain law is predominant in physics unless the same were true also in biology and psychology.

We may perhaps here deal with free will. Our consciousness, which after all is the most immediate source of cognition, assures us that free will is supreme. Yet we are forced to ask whether human will is causally determined or not. Put in this way the question, as I have frequently tried to show, is a good example of the kind of problem which I have described as illusory, by which I mean that, taken literally, it has no exact meaning. In the present instance the apparent difficulty is due to an incomplete formulation of the question. The actual facts may be briefly stated as follows. From the standpoint of an ideal and all-comprehensive spirit, human will, like every material and spiritual event, is completely determined causally. Looked at subjectively, however, the will, in so far as it looks to the future, is not causally determined, because any cognition of the subject's will itself acts causally upon the will, so that any definitive cognition of a fixed causal nexus is out of the question. In other words, we might say that looked at from outside (objectively) the will is causally determined and that looked at from inside (subjectively) it is free. There is here no contradiction, any more than there was in the previous debate about the right- and left-hand side, and those who fail to agree to this overlook or forget the fact that the subject's will is never completely subordinate to its cognition and indeed always has the last word.

In principle, therefore, we are compelled to give up the attempt to determine in advance the motives guiding our actions on purely causal lines, i.e., by means of purely scientific cognition; in other words, there is no science and no intellect capable of answering the most important of all the questions facing us in our personal life, the question, that is, how we are to act.

It might thus be inferred that science ceases to play a part as soon as ethical problems arise. Yet such an inference would be wrong. We saw above that in dealing with the structure of any science, and in discussing its most suitable arrangement, a reciprocal interconnection between epistemological judgments and judgments of value was found to arise, and that no science can be wholly disentangled from the personality of the scientist. Modern physics has given us a clear indication pointing in the same direction. It has taught us that the nature of any system cannot be discovered by dividing it into its component parts and studying each part by itself, since such a method often implies the loss of important properties of the system. We must keep our attention fixed on the whole and on the interconnection between the parts.

The same is true of our intellectual life. It is imposestablish its international validity, unlike history where it has actually been asked whether an objective history can be an ideal to be aimed at. Ethics also is supranational, otherwise ethical relations could not exist between the members of different nations. Here again physics takes up a strong position. Scientifically it is based on the principle that it must contain no contradiction, which in terms of ethics implies honesty and truthfulness; and these qualities are valid for all civilized nations and for all time; so that this scientific principle may claim to rank among the first and most important of virtues. I do not think that I exaggerate in saying that an infraction of this ethical demand is discovered and repudiated more quickly and certainly in physics than in any other science.

It is rather shocking to notice the difference between such strictness and the thoughtless laxity with which similar faults are accepted in everyday life. I have not so much in mind the so-called conventional falsehoods which in practice are harmless and to a certain extent indispensable to daily intercourse: conventional falsehoods do not deceive precisely because they are conventional. The harm begins where there is an intention to deceive the other party and to convey to him a faulty impression. It is the duty of those who work in responsible positions to reform this matter ruthlessly as well as to set an example worth following.

Justice is inseparable from truthfulness: justice, after all, simply means the consistent application in practice of the ethical judgments which we pass on opinions and actions. The laws of nature remain fixed and unchanged whether applied to great or to small phenomena, and

similarly the communal life of men requires equal rights for all, for great and small, for rich and poor. All is not well with the state if doubts arise about the certainty of the law, if rank and family are respected in the courts, if defenseless persons feel that they are no longer protected from the rapacity of powerful neighbors, and if the law is openly wrenched on grounds of so-called expediency. The populace has a keen sense of the security of the law, and nothing rendered Frederick the Great more popular than the legend of the miller of Sans Souci. Such principles made Germany and Prussia great; it is to be hoped that they will never be lost, and it is the duty of every patriot to work for their preservation and consolidation.

At the same time it must be understood that the goal at which we aim—a permanently satisfactory condition—can never be attained in its perfection. The best and maturist ethical principles must fail to take us to an ideal perfection: they can never do more than indicate the direction in which we can look for our ideal. If these facts are disregarded there is a danger that the seeker may despair altogether or may doubt the value of ethics, a state in which, especially if he is honest in his dealings with himself, he may easily end by attacking ethics. There are numerous examples of this among the philosophies of ethics. The case here is the same as in science: what is important is not to have a permanent possession, but to work unceasingly towards the ideal aim, to struggle daily and hourly toward a renewal of life, and despite every setback to strive toward improvement and perfection.

Yet in the end we may be tempted to ask whether such the real nature of causality. This method is to begin with the world of data which we possess, i.e., our experiences, to generalize, to eliminate as far as possible all anthropomorphic elements and thus cautiously to elaborate an objective concept of causality.

The many attempts which have been made in this direction show us that the best approach to the concept of causality consists in attaching it to the capacity of foretelling future events which we have acquired and tested in daily experience. And indeed there is no better means of demonstrating the causal connection between two events than to show that the occurrence of the one event can regularly permit us to forecast the occurrence of the other. This much was known to the farmer in the story who made such a striking demonstration before the skeptics of the causal connection between artificial manure and the fertility of the soil. The skeptics refused to believe that the heavy yield of clover on the farmer's field was caused by artificial manure and tried to discover some other reason. Thereupon the farmer plowed in lines having the shape of letters and had them manured while leaving the rest of his field without manure. When the clover came up in the following spring all could plainly read in letters of clover: "This part has been manured with gypsum."

I propose to commence the next stage with the simple and general proposition that an event is causally conditioned if it can be foretold with certainty. Of course I mean no more by this than that the possibility of correctly foretelling the future is a safe criterion of the presence of a causal connection; I do not mean that the two are identical. To take a familiar instance. During relatively extensive surfaces on which a very great number of molecules exert an impact; for here the irregularities cancel each other.

Variations of this kind caused by the irregular impact of molecules are observed everywhere where molecules in rapid motion are in contact with bodies easily set in motion. They can for example also be observed in the movements first described by Brown and called after him. These are the trembling movements executed by fine particles of dust suspended in a liquid and subject to the impacts of the molecules of the liquid. The fact that a very sensitive balance never attains rest but continually oscillates irregularly around the point of equilibrium, is another instance of this movement.

Various radioactive phenomena afford another example of statistical laws. A radioactive substance continuously emits a number of particles having a positive or a negative charge, a process due to the spontaneous decomposition of its atoms. When dealing with comparatively lengthy periods of time, we can fairly say that the emission is steady. When dealing with briefer periods, however, i.e., with those which do not much exceed the average interval between two consecutive emissions, we find that the process is entirely irregular.

Now the indeterminists deal with every physical law in the same way as that in which they deal with the laws of the gases and of radioactivity: they treat them as being in the last analysis a matter of contingency. For them nature is entirely a matter of statistics and it is their aim to build up physics on a calculus of probability.

In fact, however, physics has hitherto developed on

the opposite assumption, and physicists have chosen the second of the two above-mentioned alternatives. In other words, in order to be preserved intact, the principle of causality, according to which an event is causally determined only if it can be accurately foretold, has been slightly modified. What has been done is to change the sense in which the term "event" is employed. Theoretical physics does not consider an individual measurement as an event, because such a measurement always contains accidental and unessential elements. By an event, physics means a certain merely intellectual process. It substitutes a new world in place of that given to us by the senses or by the measuring instruments which are used in order to aid the senses. This other world is the so-called physical world image; it is merely an intellectual structure. To a certain extent it is arbitrary. It is a kind of model or idealization created in order to avoid the inaccuracy inherent in every measurement and to facilitate exact definition.

It follows that every measurable magnitude, every length, every period of time, every mass, and every charge, has a twofold meaning. It may be considered as the immediate result of the measurement, or it may be treated as applied to the model to which we give the name of physical world image. In the former case it can never be defined exactly, and consequently can never be represented by an exact figure; in the second case it can be denoted by definite mathematical symbols with which we can operate in accordance with exact rules. If we speak in physics of the height of a tower and use a trigonometrical equation for its calculation, we have in mind a perfectly defined magnitude; an actual measurement of the height, on the other hand, does not give us an exact magnitude. Thus the ideal height (which can always be calculated with perfect accuracy) is always something different from the actually measured height, and the same applies to the period of oscillation of a pendulum or to the brightness of an electric globe. Further, any universal constant, e.g., the velocity of light in space, or the charge of an electron, is not the same in the physical world image and in any actual measurement: in the former it is perfectly exact; in the latter it is not accurately defined. A clear and consistent distinction between the magnitudes of the world of the senses and the similarly designated magnitudes of the world image is indispensable if we wish to have a firm grasp of the matter. Without it any debate on this question will always lead to misunderstandings.

It is not therefore the case, as is sometimes stated, that the physical world image can or should contain only directly observable magnitudes. The contrary is the fact. The world image contains no observable magnitudes at all; all that it contains is symbols. More than this: it invariably contains certain components having no immediate meaning as applied to the world of the senses nor indeed any meaning at all, e.g., ether waves, partial oscillations, reference co-ordinates, etc. Such component parts may seem to be an unnecessary burden; yet they are adopted because the introduction of the world image brings with it one decisive advantage. This advantage consists in the fact that it permits a strict determinism to be carried through.

It is true that the world image fulfills no more than an auxiliary function. In the last analysis it is the events in the translation of the event from the world of the senses to the world image and back from the latter to the former for the inaccuracy inherent in forecasting an event of the sense world. It is in this that the importance of the physical world image consists.

Classical theory has tended to disregard the inaccuracies due to this transference. It has concentrated upon applying causality to the events in the world image, and by this method has obtained its striking successes. It has even succeeded in discovering a satisfactory explanation compatible with a strict causality for the above-mentioned irregular variations in the pressure of a gas, or in the movements of molecules (Brownian movement). As for the indeterminists, these phenomena do not constitute a problem for them: they look for irregularity behind every rule and statistical laws afford them immediate satisfaction. Accordingly, they confine themselves to assuming that the collision between two molecules or the impact of a single molecule on the container is governed by statistical laws. Yet there is not really any valid reason for this assumption any more than the fact that the electrons gather on the surface of a conductor allows us to infer that the charge of any individual electron is at its surface. The determinists, on the other hand, look for a rule behind every irregularity, and it is their task to formulate a theory of the laws of the gases on the assumption that the collision between any two molecules is causally determined. The solution of this problem was the lifework of the great physicist Ludwig Boltzmann, and it is one of the finest triumphs of theoretical investigation. It does not only lead to the proposition that the average energy of the oscillations and the world of sense; it is at any rate premature. It is far more natural to avoid the difficulty by another method, a method which has often rendered good services in similar cases and which consists in assuming that it is meaningless, with respect to physics, to ask for the simultaneous values of the co-ordinates and of the velocities of a material point or for the path of a photon of a given color. Evidently the law of causality cannot be blamed because it is impossible to answer a meaningless question; the blame rests with the assumptions which lead to the asking of the question, i.e., in the present case with the assumed structure of the physical world image. The classical world image has failed us and something else must be put in its place.

This has actually been done. The new world image of quantum physics is due to the desire to carry through a rigid determinism in which there is room for Planck's constant. For this purpose the material point which had hitherto been a fundamental part of the world image had to lose this supremacy. It has been analyzed into a system of material waves, and these material waves are the elements of the new world image.

The world image of quantum physics stands in approximately the same relation to classical physics as Huygens's wave optics stand to Newton's corpuscular or ray optics. The latter meets a great many instances, but it fails in others; and similarly classical or corpuscular mechanics is now seen to be no more than a special instance of the more general wave mechanics. In place of the material point of the classical system an infinitely narrow parcel of waves is found, i.e., a system of numerous waves interfering with each other in such a way as to cancel each other everywhere in space except at the place occupied by the material point.

The laws of wave mechanics differ, of course, fundamentally from those of classical mechanics with its material points. It is an essential fact, however, that the magnitude which is characteristic for the material waves is the wave function, by means of which the initial conditions and the liminal conditions are completely determined for all times and places. Definite rules of calculation are available for this purpose; it is possible to employ Schrödinger's operators, Heisenberg's matrices, or Dirac's *q* numbers.

Thus the introduction of wave functions solves the difficulty mentioned above, which arose when we asked how a single electron behaved when impinging on a crystal. The question then was whether it was reflected or penetrated the sheet. The impinging electron cannot divide into several parts; the waves, however, which are substituted for it can do so, so that interference becomes possible between the waves reflected at the front and those reflected at the back. Hitherto such a process was entirely incomprehensible: now it occurs in accordance with laws which can be exactly formulated.

We see then that there is fully as rigid a determinism in the world image of quantum physics as in that of classical physics. The only difference is that different symbols are employed and that different rules of operating obtain. Accordingly the same happens in quantum physics as we saw previously happening in classical physics. The uncertainty in forecasting events in the world of the senses disappears and in its place we have

an uncertainty with regard to the connection between the world image and the world of the senses. In other words, we have the inaccuracy arising from a transfer of the symbols of the world image to the sense world and vice versa. The fact that physicists have been willing to put up with this double inaccuracy is an impressive demonstration of the importance of maintaining the rule of determinism within the world image. At the same time a critical observer may well consider the price paid for the preservation of strict causality to be rather high. A superficial consideration shows how wide is the distance between the world image and the sense world of quantum physics, and how much more difficult it is in quantum physics to translate an event from the world image into the sense world and vice versa. Things are no longer as simple as they were in classical physics. There the meaning of each symbol was entirely clear; the position, the velocity, and the energy of a material point could be established more or less directly by measurement, and there was no apparent reason why it should not be assumed that any remaining inaccuracy would eventually be reduced below any given limit in the course of the progressively growing accuracy of the methods of measurement. The wave function of quantum mechanics, on the other hand, affords us in the first instance no help at all for an interpretation of the world of the senses; and while the term wave is expressive and suitable, it must not be allowed to disguise the fact that its meaning in quantum physics is totally different from that which it formerly had in classical physics. In classical physics a wave is a definite physical process, a movement perceptible by the senses or an alternating electrical field admitting of direct measurements, whereas in quantum physics it really denotes no more than the probability that a certain state exists. When a photon or electron impinges on the sheet of crystal it is not these entities which are divided, and thus lead to the phenomena of interference; all that we have is the probability that the indivisible photon or electron is present. It is only when a vast number of photons or electrons are impinging that this magnitude denotes a perfectly definite number of photons or electrons.

Such considerations have caused the indeterminists to renew their attacks on the law of causality. In the present instance they have some reason for expecting a certain positive success, since all measurements must have a merely statistical significance so far as they relate to wave functions. Yet here again the champions of strict causality have the same means of escape as before. Once again they can assume that there is no definite meaning in inquiring after the significance of any given symbol of the world image of quantum physics (e.g., a material wave) unless it is stated at the same time how this significance is to be determined and what is the condition of the special measuring instrument used in order to apply the symbol to the world of the senses. It is customary for this reason to speak of the causal work of the measuring instrument employed, by which it is meant that the inaccuracy is due at any rate in part to the fact that the magnitude to be measured is connected by some kind of law with the means by which it is measured.

As a matter of fact, every measurement, whatever the method of its employment, invariably interferes more

or less with the event to be measured, as was seen above when we dealt with the electron in motion whose path is interfered with when it is illuminated, the interference varying with the intensity of the illumination, and the illumination being essential for the measurement. Accordingly, when a given material wave at various times corresponds to various events in the world of the senses, the reason is that the sensuous meaning of the material wave does not depend solely upon the wave itself but also depends on the reciprocal interference between the wave and the measuring instrument.

The above assumption gives a new development to the entire question, the further course of which is as yet uncertain. For now the indeterminists can fairly ask whether the concept of the causal influence exerted by the measuring instrument upon the measured event has any rational meaning at all, in view of the fact that we are acquainted with the event only by measuring it, so that every measurement brings about a fresh causal interference—in other words, a fresh disturbance of the event. Thus it looks as though it must be impossible to distinguish between the "event in itself" and the apparatus by which it is measured.

This objection does not, however, meet the case. Every experimental physicist is aware that there are indirect as well as direct methods and that in many instances where the latter failed, the former have rendered useful services. And it is even more important that a word should be said to refute a widespread and plausible opinion which holds that a problem in physics deserves to be examined only when it is certain in advance that it admits of a definite answer. If this rule had always been followed, the famous experiment made by Michelson and Morley in order to measure the so-called absolute velocity of the earth would never have been undertaken, and we might well be without the theory of relativity today. The problem of the earth's absolute velocity has for some time been seen to be somewhat insignificant; yet the trouble spent upon it has proved extremely useful for physics. It is all the more likely that it may prove worth while to pursue the problem of a strict causality, since this question is far from being settled and might prove more fruitful than any other question in physics.

The question then remains how we are to reach a decision. Clearly all that we can do is to adopt one of the two opposite views and to see whether it leads to useless or to fruitful results. To this extent it is satisfactory to see that the physicists who interest themselves in this problem tend to fall into two schools, one of which tends toward determinism while the other tends toward indeterminism. It would seem that at present the latter constitute the majority, although it is not easy to be certain and changes may well occur in course of time. There might also be room for a third party which might take up a kind of mediating position, treating certain concepts like those of electrical attraction, or gravitation, as possessing an immediate significance and as being subject to strict laws while assuming others, like those of the light wave or material wave, to have a merely statistical meaning for the world of the senses. Yet such a view might be considered unsatisfactory because of its lack of unity, so that for the moment I propose to leave

it aside and to deal with the two completely consistent points of view.

When the indeterminist finds that the wave functions of quantum physics are simply statistical magnitudes his zeal is satisfied and he feels no impulse to ask further questions. Again, when dealing with radioactive processes, he is satisfied to find, e.g., that a given number of atoms of any radium combination decompose on an average per second, and he does not ask why one atom happens to be decomposing now while its neighbor may survive a thousand years. On the other hand, a definite natural law like Coulomb's law of electrical attraction is an unsolved problem for him since he cannot rest satisfied with Coulomb's method of expressing the potential and is compelled to look for exceptions. He rests satisfied only when he has succeeded in establishing the degree of probability that the electrical force differs from Coulomb's value by a certain given amount.

The determinist's standpoint is diametrically opposite in each detail; he is satisfied with Coulomb's law of electrical attraction because it is entirely definite, but he recognizes the wave functions as magnitudes having a probable value only so long as the apparatus is disregarded by which the wave is produced or analyzed. Further, he looks for a strict law governing the relations between the properties of the wave functions and the events in the bodies standing in a relation of reciprocal causality with the wave. For this purpose he must, of course, study all these bodies as well as the wave function, and he must transfer not only the entire experimental apparatus used for the production of the material waves—high-tension battery, incandescent wire, and radioactive material—but also the measuring apparatus, the photographic plate, the ionization chamber, and Geiger's counter together with all the events occurring therein into his physical world image: and he must treat all these objects as constituting one single field of study, as a complete totality.

Of course this does not constitute a settlement of the problem; the problem, on the contrary, has for the moment become all the more complicated. It is not permissible to cut the structure in pieces, nor is any external interference permitted under penalty of destroying its uniqueness, so that a direct study of it is altogether impossible. On the other hand, we are now in a position to make certain novel hypotheses with regard to the internal events and subsequently to check the consequences. The future will show whether any advance is possible on these lines, and at the moment we cannot clearly see in what direction the advance is likely to lead. It may, however, be regarded as certain that Planck's constant constitutes an objective limit beyond which the physical measuring instruments we possess cannot reach, and which will prevent us for all time from understanding the full causality of the most delicate physical processes "in themselves," i.e., apart from their origin and their effects.

In a way it would seem that we have now reached the end of our consideration, in the course of which we found that a strictly causal way of looking at things—"causal" being taken in the modified sense explained above—is wholly compatible with modern physics although its necessity cannot be demonstrated either a priori or a posteriori. Yet even here an objection occurs calculated to prevent a convinced determinist from being entirely satisfied with the interpretation of causality here introduced. Indeed, the objection is more likely to appeal to a determinist than to other persons. Even though we should succeed in developing the concept of causality on the lines here described, it will permanently be vitiated by a grave and fundamental defect. We were enabled to carry through the determinist view of the universe only by substituting the physical world image for the immediate world of the senses. Now the world image is due to our imagination and is of a provisional and changeable character; it is an emergency concept, hardly worthy of a fundamental physical notion, and the question arises whether it might be possible to endow the concept of causality with a more deep and direct significance by making it independent of the introduction of an artificial human product. This could be done by applying it, not to the physical world image but immediately to the experiences of the world of the senses. We shall, of course, have to maintain our original proposition, to the effect that an event is causally determined if we can accurately predict it. Otherwise we would be surrendering our principle, which was to begin solely from actual experience. At the same time we are also compelled to accept our second proposition, which was that it was never possible to predict any event. It follows, in the same way as we saw above, that the first proposition must be somewhat modified if we wish to retain causality in nature. So far everything remains unchanged. The possibility now, however, arises of substituting a different and in a sense a contrary modification for the one hitherto adopted.

What we modified above was the *object* of the prediction, i.e., the event. What we did there was to refer the events not to the immediately given world of the senses, but to a fictitious world image, by which process we were enabled to achieve an exact determination of the events. Now it is equally possible to modify the *subject* of the prediction, i.e., the predicting intellect. Every prediction implies a predicting person. In the subsequent argument I propose to concentrate upon the predicting subject and to treat the immediately given events of the world of the senses as object. An artificial world image will not be introduced at all.

It is easy to appreciate that the accuracy of a prediction largely depends on the individuality of the predictor. To revert to a weather forecast: it makes all the difference whether tomorrow's weather is foretold by somebody who knows nothing about the atmospheric pressure, the direction of the wind, and the moisture and temperature of the air, or by a practical farmer who notes all these things and has a long experience beside, or finally by a trained meteorologist who has weather charts from every part of the world, with exact data apart from this local information. The forecasts made by this series of prophets will show a diminishing degree of inaccuracy. That being so, we are induced to assume that an ideal intellect having complete knowledge of today's physical events in all places should be in a position to foretell tomorrow's weather with complete accu-

racy. The same applies to every forecast of physical events.

Such an assumption implies an extrapolation, a generalization which can neither be proved nor disproved by logical processes, and which consequently can be judged, not in accordance with its truth, but only in accordance with its value. From this point of view the impossibility of foretelling an event with complete accuracy in any single instance, whether we assume the standpoint of classical or quantum physics, appears to be the natural consequence of the fact that man with his senses and his apparatus is himself a part of nature to whose laws he is subjected. An ideal intellect is not so bound.

It might be objected that this ideal intellect itself is only a product of our thoughts and that the thinking brain is composed of atoms obeying physical laws. This objection will not bear close investigation. It is certain that our thoughts can carry us beyond any natural law known to us and that we can imagine connections between events which go far beyond those obtaining in physics. If it is claimed that the ideal intellect can exist only in the human brain, and would vanish with the disappearance of the latter, then in order to be consistent it would also have to be claimed that the sun and the whole external world in general can only exist in our senses, since these are the only source of scientific cognition. Yet every reasonable person must be convinced that the sun's light would not be diminished in the least even if the whole of mankind were to perish.

For we must take care not to regard the ideal spirit as ranking with ourselves; we have no right to ask it how it acquires the knowledge enabling it to foretell exactly future events, since such inquisitiveness might well meet with the reply: "You resemble the spirit which you can grasp, you do not resemble me." If the inquirer should remain obstinate despite this answer and should insist that the notion of an ideal spirit if not illogical, is at any rate void of content and superfluous, then we may fairly reply that a proposition is not scientifically valueless merely because it lacks logical foundation, and that such narrow formalism obstructs the source from which men like Galileo, Kepler, Newton, and many other great physicists drew their scientific inspiration. Consciously or unconsciously a devotion to science was a matter of faith for these men; they had an unshakeable faith in a rational order of the world.

At the same time such a belief is not compulsory: we cannot order men to see the truth or prohibit them from indulging in error. Yet the simple fact that we are enabled, if only to a limited extent, to subject future natural events to our intellectual operations, and to guide them in accordance with our will, would necessarily remain a wholly unintelligible mystery if it did not allow us to have, at any rate, a premonition of a certain harmony between the outer world and the human spirit. Logically the extent which we attribute to the realm of this harmony is a question of secondary importance. The most perfect harmony, and consequently the strictest causality in any case, culminates in the assumption that there is an ideal spirit having a full knowledge of the action of the natural forces as well as of the events in the intellectual life of men; a knowledge extending to every detail and embracing present, past, and future.

cause every application of the law of causality to the will of the individual and every information gained in this way is itself a motive acting upon the will, so that the result which is being looked for is continually being changed. Hence it would be a complete mistake to attribute the impossibility of forecasting the subject's actions on purely causal lines to a lack of knowledge which might be overcome if the individual intelligence were suitably increased. Such an inference is analogous to the process of ascribing the impossibility of simultaneously determining exactly the position and the velocity of an electron to the inadequacy of our methods of measuring. The impossibility of foretelling the subject's actions on purely causal lines is not based on any lack of knowledge, but on the simple fact that no method by whose application the object is essentially altered can be suitable for the study of this object.

In consequence intellectual man can never have recourse to the principle of causality to determine his acts of will; for this purpose he must refer to a totally different law, namely, the law of ethics, which is based on a different foundation and cannot be comprehended solely by scientific methods.

Scientific thought always requires a certain distance and a clear separation as between the thinking subject and the object of his thought, and this distance is best guaranteed by the assumption of an ideal spirit. Now such a spirit can only be subject and can never be object.

It may be said that it constitutes an unsatisfactory negation if we are prohibited from making the ideal spirit the object of our thoughts; and it may be added that this may be too high a price to pay for a rigorous possible to continue to eliminate the question of causality in the sphere of the natural sciences.

It is true that the law of causality cannot be demonstrated any more than it can be logically refuted: it is neither correct nor incorrect; it is a heuristic principle; it points the way, and in my opinion it is the most valuable pointer that we possess in order to find a path through the confusion of events, and in order to know in what direction scientific investigation must proceed so that it shall reach useful results. The law of causality lays hold of the awakening soul of the child and compels it continually to ask why; it accompanies the scientist through the whole course of his life and continually places new problems before him. Science does not mean an idle resting upon a body of certain knowledge; it means unresting endeavor and continually progressing development toward an aim which the poetic intuition may apprehend, but which the intellect can never fully grasp.

to differ, it is only because they have to be adapted to the different subjects which they treat. This inner resemblance has become more and more evident in recent times, to the great advantage of the whole of science. Hence I consider myself entitled to begin with considerations applying to the whole of science; although of course when I pass to more particular applications I shall tend to confine myself to my own subjects.

Let me begin by asking how a scientific idea arises and what are its characteristics. In asking these questions I cannot attempt, of course, to analyze the delicate mental processes taking place in the investigator's mind and what is more, largely in his subconscious mind. These processes are mysteries which can be revealed only to a limited extent if at all, and it would be equally foolish and rash to attempt any study of their inmost nature. The most that we can do is to begin with the obvious facts, which means that we investigate those ideas which have actually proved their leavening force for any branch of science; and this in turn means that we ask in what form they first occurred and what was their content at that time.

The first result of such an investigation is the discovery of the following rule: any scientific idea arising in the mind of a scholar is based on a concrete experience, a discovery, an observation, or a fact of any kind, whether it is a physical or an astronomical measurement, a chemical or a biological observation, a discovery among the archives or the excavation of some valuable relic of an earlier civilization. The content of the idea consists in this experience being compared and being brought into contact with certain different experiences

in the mind of the scholar, in other words, in the fact that it establishes a link between the old and the new, so that a number of facts which had hitherto co-existed loosely are now definitely interrelated. The idea becomes fruitful and hence attains value for science if the interconnection thus established can be applied more generally to a series of cognate facts: for the establishment of an interconnection creates order, and order simplifies and perfects the scientific view of the universe. What is most important, however, is that the task of applying the new idea in its entirety shall lead to new questions and hence to new studies and to new successes. And this is true of the physicist's hypotheses no less than of the interpretations established by the philologist.

I propose now to exemplify the above in some detail, and in doing so I desire to confine myself to my own subject of physics. The angle of vision may appear somewhat restricted; on the other hand, I shall be able to throw a clearer light upon the subject.

A classical example of the sudden emergence of a great scientific idea is found in the story of Sir Isaac Newton who, sitting under an apple tree, was reminded by a falling apple of the movement of the moon around the earth and thus connected the acceleration of the apple with that of the moon. The fact that these two accelerations are to each other as the square of the radius of the moon's orbit is to the square of the earth's radius, suggested to him the idea that the two accelerations might have a common cause and thus provided him with a foundation for his theory of gravitation.

Similarly, James Clerk Maxwell, on comparing the strength of a current measured electromagnetically with passing, was considered irrelevant by the energetist school and was passed over in silence.

I myself experienced during the eighties and nineties of the last century what are the feelings of a student who is convinced that he is in possession of an idea which is in fact superior, and who discovers that all the excellent arguments advanced by him are disregarded simply because his voice is not powerful enough to draw the attention of the scientific world. Men having the authority of Wilhelm Ostwald, Georg Helm, and Ernst Mach were simply above argument.

The change originated from a different side altogether: atomism began to make itself felt. The atomic idea is extremely old; but its first adequate formulation took shape in the kinetic gas theory which originated more or less contemporaneously with the discovery of the mechanical heat equivalent. The energists at first opposed it vigorously, and it led a modest existence; toward the end of last century, however, experimental investigation led to its rapid success. According to the atomist idea the transference of heat from the hotter to the colder body does not resemble the falling of a weight; what it resembles is a mixing process, as when two different kinds of powder in a vessel, having first constituted different layers, eventually mingle with each other if the vessel is continually shaken. If this happens the powder does not oscillate between a state of complete mixture and complete isolation of the constituent powders; what happens is that the change takes place once in a certain sense, viz., in the direction toward complete mixture, and is then at an end: the process is an irreversible one. Seen in this light the second principle of

thermodynamics is found to be of a statistical nature: it states a probability: the arguments supporting this view and indeed raising it beyond any doubt have been well stated by my colleague Max von Laue.

The historical development here described may well serve to exemplify a fact which at first sight might appear somewhat strange. An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that Saul becomes Paul. What does happen is that its opponents gradually die out and that the growing generation is familiarized with the idea from the beginning: another instance of the fact that the future lies with youth. For this reason a suitable planning of school teaching is one of the most important conditions of progress in science. Accordingly, I should like here briefly to deal with this point.

What is learned at school is not as important as how it is learned. A single mathematical proposition which is really understood by a scholar is of greater value than ten formulas which he has learned by heart and even knows how to apply, without, however, having grasped their real meaning. The function of a school is not so much to teach a businesslike routine as to inculcate logical and methodical thought. It may be objected that ultimately it is the ability to do things rather than knowledge that matters; and it is true that the latter is valueless without the former, just as any theory is ultimately important only by reason of its particular applications. Yet routine can never be a substitute for theory, for in any cases that fall outside the rule, routine breaks down. Hence the first requisite, if good work is to be done, is a thorough elementary training; and here it is not so much the quantity of facts learned as the manner of treatment that matters. Unless this preliminary training is acquired at school, it is hard to obtain it at a later stage: training colleges and universities have other tasks. For the rest, the last and highest aim of education is neither knowledge nor the ability to do things, but practical action. Now practical action must be preceded by the ability to act, and the latter in turn demands knowledge and understanding. The present age, which lives at such a rapid rate, and shows so much interest for every innovation having an immediate sensational effect, provides us with instances where scientific training tends to anticipate certain exciting results before they have properly ripened; for the public is favorably impressed if the curriculum of an intermediate school already contains modern problems of scientific investigation. Yet such a practice is exceedingly dangerous. The problems cannot possibly be dealt with thoroughly, and the consequence may easily be to induce a certain intellectual superficiality and empty pride in knowledge. I should consider it extremely dangerous if the intermediate schools were to deal with the theory of relativity of the quantum theory. Specially gifted scholars always require exceptional treatment; but the curriculum is not designed for such, and I would definitely condemn any attempt to take such a question as that of the universal validity of the principle of the preservation of energy—which, of course, today is seriously regarded as an open one in nuclear physics—and to treat it as debatable before pupils who cannot have properly grasped the meaning of the principle involved, much less its potential scope.

The results of such an up-to-the-minute method of teaching become all too plain when we consider the way in which the breakdown of the exact sciences is occasionally spoken of today. It is characteristic of the prevalent confusion that there are numbers of inventive minds busying themselves today upon devices which aim at the unlimited production of energy of the utilization of the fashionable mysterious earth rays. And it is even more surprising that credulous persons provide ample funds for such inventors, while really valuable and hopeful scientific investigations are hampered or actually stopped by lack of means. A thorough school training might here prove a useful remedy, and this would apply to the patrons no less than to the inventors.

After this educational digression I should like briefly to deal with another physical idea whose varying fate may prove even more instructive than the changes undergone by the theory of heat. What I have now in mind is the idea of the nature of light.

The study of the nature of light began with the measurements of the speed of light. The idea which led Newton to his emanation theory established a comparison between a ray of light and a jet of water; the velocity of light was compared with the velocity of particles of water flying in a straight line. This hypothesis, however, failed to give an account of the phenomenon of light interference, i.e., of the fact that two rays of light meeting at a point can in certain circumstances produce darkness at this point. Accordingly the emanation theory was